Revision for Algebra and Statistics

First: Algebra.

The Cartesian product of two finite sets and representing it.

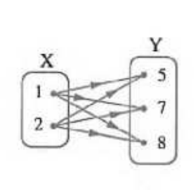
If $X = \{1,2\}$, $Y = \{5,7,8\}$, then :



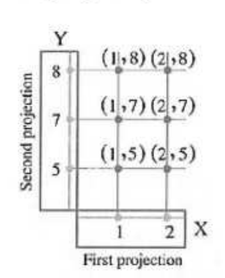
X × Y

is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y.

i.e. $X \times Y = \left\{ \begin{matrix} (1,5), (1,7), (1,8), \\ (2,5), (2,7), (2,8) \end{matrix} \right\}$



The arrow diagram

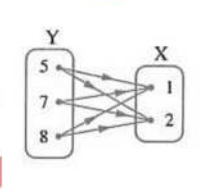


The graphical diagram (The Cartesian diagram)

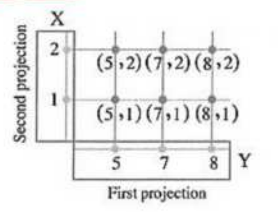
Y × X

is the set of all ordered pairs whose first projection of each of them belongs to Y and the second projection of each of them belongs to X.

i.e. $Y \times X = \left\{ \begin{matrix} (5,1), (5,2), (7,1), \\ (7,2), (8,1), (8,2) \end{matrix} \right\}$



The arrow diagram

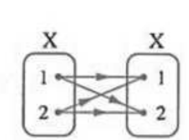


The graphical diagram (The Cartesian diagram)

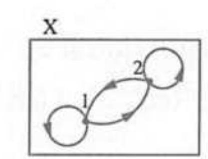
X × X

is the set of all ordered pairs whose first projections and second projections belong to X.

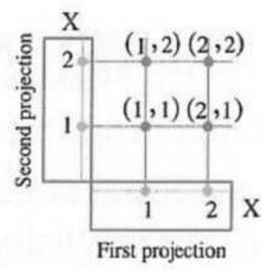
i.e. $X \times X = \{(1,1), (1,2), (2,1), (2,2)\}$



The arrow diagram



The arrow diagram



The graphical diagram (The Cartesian diagram)

Remarks.

(1) $X \times Y \neq Y \times X$, where $X \neq Y$

(2) $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$ where n is the number of elements

(3) $n(X \times X) = n(X^2) = [n(X)]^2$ (4) $X \times \varnothing = \varnothing \times X = \varnothing$

The relation and its representing

•*The relation from the set X to the set Y is a connecting joining some or all the elements of X with some or all the elements of Y.*

If R is a relation from the set X to the set Y, then:

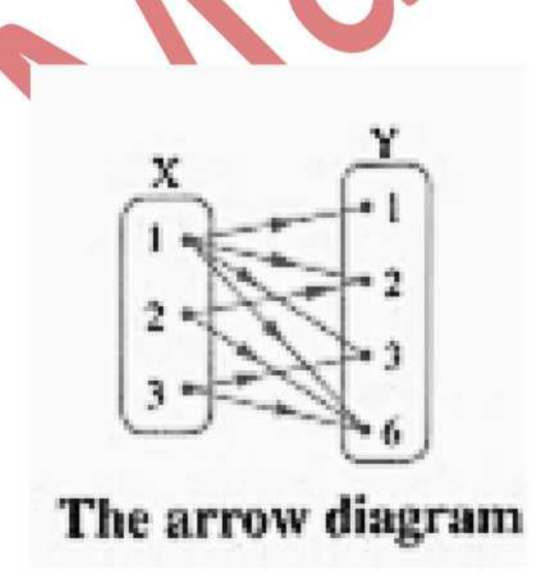
1. *R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y.*
2. *$R \subset X \times Y$*
3. *The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically) If R is a relation from X to X, then R is a relation on X and $R \subset X \times X$*

Example:

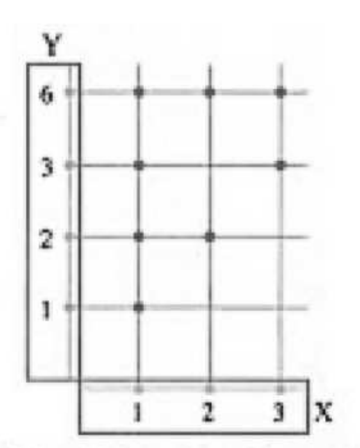
If X = {1,2,3} , Y ={1,2,3,6} and R is a relation from X to Y where "a R b" means "a is a factor of b" for each a ∈ X ,b ∈ Y then write R and represent it by an arrow diagram and a Cartesian diagram.

Solution

R = {(1,1) , (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6)}



The arrow diagram



The Cartesian diagram

The function

A relation from X to Y is said to be a function if :

1. *Each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.*

2. *Each element of the set X has one and only one arrow going out of it to one element of Y in the arrow diagram which represents the relation.*

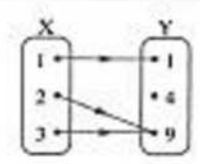
3. *Each vertical line has one and only one point lying on it of the points which represent the relation in the Cartesian diagram which represents the relation.*

If f is a function from the set X to the set Y is written as f: X → Y, then :

1- *X is called the domain of the function f*
2- *Y is called the codomain of the function f*
3- *The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y.*

For example

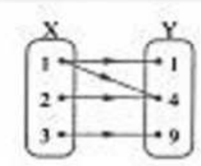
If X = {1,2,3} , Y = {1,4,9} then the following diagrams show some of the relations from X to Y and we note which of the following relations represent a function from X to Y and which does not represent:



Note : Going out only one arrow from each element of the elements of X

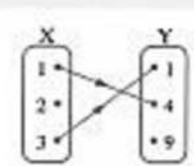
Then : The relation is a function from X to Y

- The domain = {1 , 2 , 3}
- The range = {1 , 9}



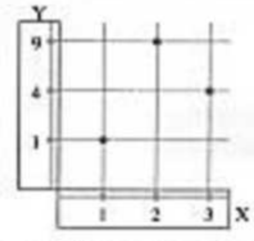
Note : Going out two arrows from the element 1 in X

Then : The relation is not a function from X to Y



Note : There are not arrows going out from the element 2 in X

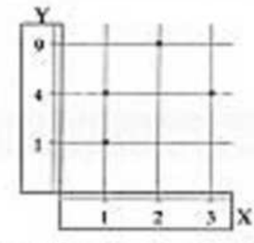
Then : The relation is not a function from X to Y



Note : Each vertical line has only one point lying on it

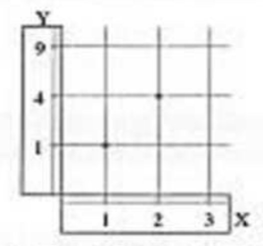
Then : The relation is a function from X to Y

- The domain = {1 , 2 , 3}
- The range = {1 , 4 , 9}



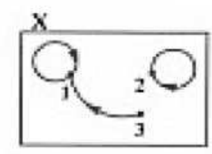
Note : There are two points lying on the vertical line at the element 1 in X

Then : The relation is not a function from X to Y



Note : There is not a point lying on the vertical line at the element 3 in X

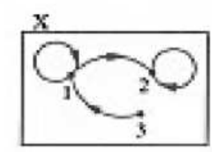
Then : The relation is not a function from X to Y



Note : Going out only one arrow from each element of the elements of X

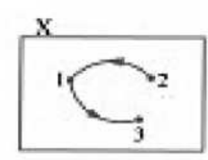
Then : The relation is a function on X

- The domain = {1 , 2 , 3}
- The range = {1 , 2}



Note : Going out two arrows from the element 1 in X

Then : The relation is not a function on X



Note : There are not arrows going out from the element 3 in X

Then : The relation is not a function on X

The polynomial functions

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified:

1- Each of the domain and the codomain of the function is the set of real numbers.

2- The power (The index) of the variable X in any of its terms is a natural number with noticing that the degree of the function is the highest power of the variable X.

For example:

The function f: f(x) = 3 is a polynomial function of zero degree.

The function f: f(x) = 2X+1 is a polynomial function of the first degree.

The function f: $f(x) = x^3 - 5x^2 + 1$ is a polynomial function of the third degree.

While :

The function f : $f(x) = \frac{1}{x^2} + x^2$ is not a polynomial function because : $\frac{1}{x^2} = x^{-2}$

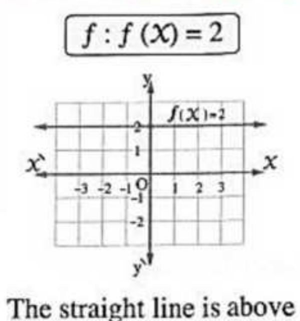
i.e. The index of the symbol X is not a natural number.

The graphical representation of the polynomial function.

The constant function

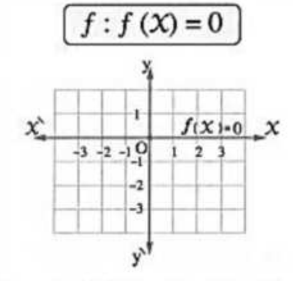
The function f: R ⟶ R where f (X) = b , b ∈ R is represented by a straight line parallel to X-axis and intersects y-axis at the point (0, b)

f : f (X) = 2



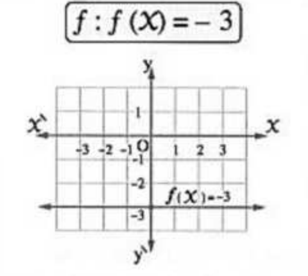
The straight line is above X-axis and passes through the point (0 , 2) (is of zero degree)

f : f (X) = 0



The straight line is coincident with X-axis and passes through the point (0 , 0) (has not degree)

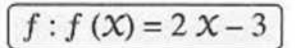
f : f (X) = – 3



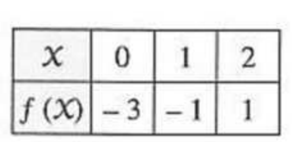
The straight line is below X-axis and passes through the point (0 , – 3) (is of zero degree)

The linear function

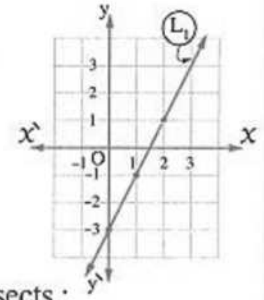
The function f: R ⟶ R where $f(x) = ax+b$, $a \in R-\{0\}$, $b \in R$ is called a linear function (function of the first degree) and is represented by a straight line intersecting y-axis at (0, b) and X-axis at $(\frac{-b}{a}, 0)$.



$f: f(x) = 2x - 3$



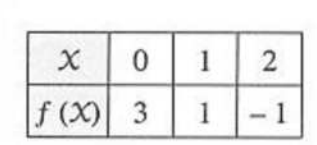
x	0	1	2
$f(x)$	−3	−1	1



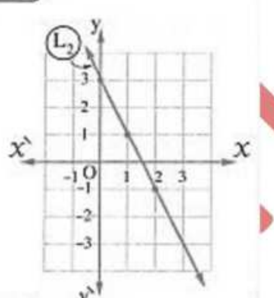
The straight line L_1 intersects :

- x-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , – 3)

$f: f(x) = 3 - 2x$



x	0	1	2
$f(x)$	3	1	−1



The straight line L_2 intersects :

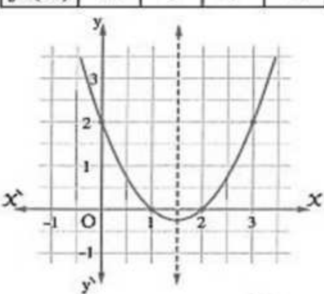
- x-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , 3)

The quadratic function

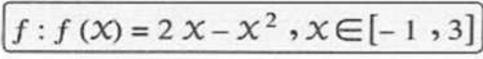
The function f: R ⟶ R where $f(x) = ax^2+bx+c$, a , b and $c \in R$, $a \neq 0$ is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$f: f(x) = x^2 - 3x + 2, x \in [0, 3]$

x	0	1	2	3
$f(x)$	2	0	0	2

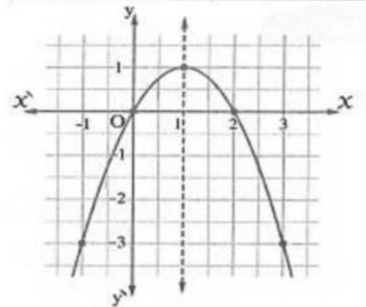


- The vertex of the curve = $\left(\frac{3}{2}, -\frac{1}{4}\right)$
- The minimum value of the function = $-\frac{1}{4}$
- The equation of line of symmetry : $x = \frac{3}{2}$



$f: f(x) = 2x - x^2, x \in [-1, 3]$

x	−1	0	1	2	3
$f(x)$	−3	0	1	0	−3



- The vertex of the curve = (1 , 1)
- The maximum value of the function = 1
- The equation of line of symmetry : $x = 1$

The ratio and its properties.

- *The ratio between the two real numbers a and b is written as $\frac{a}{b}$ or a:b and a is called the antecedent of the ratio, b is called the consequent and a, b are called the two terms of the ratio.*

- *The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.*

- *The value of the ratio changes if we add or subtract (to or from) each of its two terms the same non-zero real number.*

- *If the ratio between two numbers is ab, then: The first number = am , The second number = bm, m ≠0*

Example

Two numbers, their sum is 28 and the ratio between them is 3:4, what are the two numbers?

Solution

Let the two numbers be 3 m, 4 m

3 m + 4 m = 28 7 m = 28 m= 4

The two numbers are: 3 x 4 and 4 x 4

i.e. 12 and 16 .

The proportion

The proportion is the equality of two ratios or more.

If $\frac{a}{b} = \frac{c}{d}$ then a, b, c and d are proportional quantities.

If a, b, c and d are proportional quantities, then $\frac{a}{b} = \frac{c}{d}$

The properties of the proportion.

Property 1 :

If $\frac{a}{b} = \frac{c}{d}$ then a x d=b x c

i.e. the product of the extremes = the product of the means.

Example Find the fourth proportional of the quantities : 3,4 and 27

Let the fourth proportional be X

The quantities: 3,4,27 and X are proportional

$\frac{3}{4} = \frac{27}{x}$ 3 × X = 4 x 27

The fourth proportional = 36.

Property 2 :

If a x d= b x c, then $\frac{a}{b} = \frac{c}{d}$

Also, each of the following proportions is correct:

$\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, $\frac{b}{a} = \frac{d}{c}$.

Example

If $\frac{x+3y}{2x-y} = \frac{4}{3}$ then find the ratio X: y.

$\frac{x+3y}{2x-y} = \frac{4}{3}$, 3 (X+3 y) = 4 (2 x - y)

3X + 9y = 8X - 4y , 13 y = 5 x , X: y = 13:5

Property 3 :

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{The\ antecedent\ of\ the\ first\ ratio}{The\ antecedent\ of\ the\ second\ ratio} = \frac{The\ consequent\ of\ the\ first\ ratio}{The\ consequent\ of\ the\ second\ ratio}$

For example:

If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ or $\frac{b}{a} = \frac{3}{4}$

Property 4:

If $\frac{a}{b} = \frac{c}{d}$, then a = cm, b = dm where m is a constant ≠ 0.

Example

If a: b=3:5 , then find the ratio 20 a -7b : 15 a+ b.

$\frac{a}{b} = \frac{3}{5}$

a= 3 m , b=5 m where m ≠0

Substituting by a and b in terms of m

$\frac{20\ a-7b}{15\ a+b} = \frac{60\ m-35\ m}{45m+5\ m} = \frac{25\ m}{50\ m} = \frac{1}{2}$.

Remark

If a, b, c and d are proportional quantities and we assume that : $\frac{a}{b}=\frac{c}{d}=m$

then a = bm , c = dm

For example:

If $\frac{a}{b}=\frac{c}{d}=\frac{3}{4}$ *then* $a=\frac{3}{4}b$, $c=\frac{3}{4}d$

Generally : If a,b,c,d,e,f,... are proportional quantities and we assume that:

$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\cdots=m$, *then a = bm, c= dm , e = fm ,..*

Example.

If a,b,c and d are proportional quantities, prove that :

1- $\frac{2a+3c}{7a-5c}=\frac{2b+3d}{7b-5d}$

2- $\frac{a+c}{b+d}=\frac{a^2+c^2}{ab+cd}$

Solution

Let $\frac{a}{b}=\frac{c}{d}=m$, $a=bm$, $c=dm$

1- L.H.S. $=\frac{2\,bm+3\,dm}{7\,bm-5dm}=\frac{m\,(2b+3d)}{m\,(7b-5d)}=\frac{2b+3d}{7b-5d}=$ R.H.S

2- $\frac{a+c}{b+d}=\frac{bm+dm}{b+d}=\frac{m\,(b+d)}{b+d}=m$ (1)

$\frac{a^2+c^2}{ab+cd}=\frac{(bm)^2+(dm)^2}{bm\times b+dm\times d}=\frac{b^2m^2+d^2m^2}{b^2m+d^2m}=\frac{m^2\,(b^2+d^2)}{m\,(b^2+d^2)}=m$ (2)

From (1) and (2), we deduce that : $\frac{a+c}{b+d}=\frac{a^2+c^2}{ab+cd}$

Property 5 :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m, ... Are non-zero real numbers,

Then $\frac{m_1\, a+m_2c+m_3e+\cdots}{m_1\, b+m_2d+m_3f+\cdots}$ =one of given ratios

Example :

If $\frac{a+3b}{x+5y} = \frac{3b+5c}{5y+7z} = \frac{5c+a}{7z+x}$, prove that : $\frac{a}{3b} = \frac{x}{5y}$.

Solution

Multiplying the two terms of 2nd ratio by (-1) and adding the antecedents and consequents of the three ratios:

$\therefore \frac{a+3b-3b-5c+5c-a}{x+5y+5y+7z-7z-x} = \frac{2a}{2x} = \frac{a}{x}$ = one of the given ratios. (1)

Multiplying the two terms of 3rd ratio by (-1) and adding the antecedents and consequents of the three ratios :

$\frac{a+3b+3+5c-5c-a}{x+5y+5y+7z-7z-x} = \frac{6b}{10} = \frac{3b}{5y}$ = one of the given ratios (2)

From (1) and (2), we deduce that : $\frac{a}{x} = \frac{3b}{5y}$ $\therefore \frac{a}{3b} = \frac{x}{5y}$.

The continued proportion :

The quantities a, b and c are said to be in continued proportion if : $\frac{a}{b} = \frac{b}{c}$

a is called the first proportional, c is called the third proportional and b is called the middle proportional (proportional mean).

$\because \frac{a}{b} = \frac{b}{c}$ $\therefore b^2 = ac$ $\therefore b = \pm\sqrt{ac}$

The middle proportional between two quantities = $\pm\sqrt{\text{the product of the two quantities.}}$

Notice that :

The two quantities a and c should be either positive together or negative together.

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ = m, then 1) c =dm

2) b = dm^2

3) a =dm^3.

Example :

If a,b,c and d are in continued proportion, then prove that : $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$.

Solution

$\because$ a,b,c,d are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore$ c =dm , b = dm^2 , a =dm^3.

$\therefore \frac{2a+3c}{2b+3d} = \frac{2dm^3+3dm}{2dm^2+3d} = \frac{dm\,(2m^2+3)}{d(2m^2+3)} = m$ (1)

, $\frac{a-c}{b-d} = \frac{dm^3-dm}{dm^2-d} = \frac{dm\,(m^2-1)}{d(m^2-1)} = m$ (2)

From (1) and (2), we deduce that : $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$

The direct variation and inverse variation.

Direct variation	Inverse variation
If y varies directly as X and is written as $y \alpha x$, then : 1) y=mx (i.e. $\frac{y}{x}$ =m) where m is constant ≠ 0 2) $\frac{y_1}{y_2} = \frac{x_1}{x_2}$. 3) The relation between X and y is represented graphically by a straight line passing through the origin point. To prove that $y \alpha x$, we prove that: y = m X where m is a constant ≠0. For example: If y = 5 X, then $y \alpha x$	If y varies inversely as X 1 and is written as $y\alpha \frac{1}{x}$, then : 1) Y = $\frac{m}{x}$ (i.e. xy =m). where m is constant ≠ 0 2) $\frac{y_1}{y_2} = \frac{x_2}{x_1}$. 3) The relation between X and y is not a linear relation. To prove $y \alpha \frac{1}{x}$, we prove that: X y = m where m is a constant ≠ 0 For example: If y = $\frac{7}{x}$, then xy=7 and then $y \alpha \frac{1}{x}$
Example on direct variation 1) If $a \alpha b$, a=5 when b=2 , find a when b=3 2) If $a^2 + 4b^2$= 4ab , prove that : $a\alpha b$ Solution 1) $\because a\alpha b$ $\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$ $\therefore \frac{5}{a_2} = \frac{2}{3}$ $\therefore a_2 = 7$ 2) $\because a^2 + 4b^2$= 4ab $\therefore a^2 - 4ab + 4b^2$= 0 $\therefore (a - 2b)^2$= 0 ∴ a-2b = 0 $\therefore a = 2b$ $\therefore a \alpha b$	Example on inverse variation If x and y are two real variables where : $x^2y^2 + 25 = 10\,xy$, prove that : X varies inversely as y Solution $\because x^2y^2 - 10\,xy + 25 = 0$ $\therefore (xy - 5)^2$=0 ∴ xy-5 =0 ∴xy = 5 $\therefore x \alpha \frac{1}{y}$

Statistics

The resources of collecting data.

1) Primary resources (field resources).
2) Secondary resources (historical resources).

The methods of collecting data

1) Method of mass population
2) Method of samples

1- Choose the correct answer from those given.

1. If: (x-1 , 13) = (8 , y-3) , then $\sqrt{x+y}$ = ...

a) $\sqrt{5}$ **b)** 5 **c)** 7 **d)** 25

2. If: $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{a+b+c}{3x}$, then x = ...

a) 3 **b)** 9 **c)** 4 **d)** 2

3. The middle proportional between 4 and 9 is ....

a) 6

b) -6

c) ± 6

d) 36

4. If: n(x) = 3 , n(x , y) = 12 , then n(y) = ...

a) 4

b) 9

c) 15

d) 36

5. The constant function f: f(x) = 3 is represented graphically by a straight line that ....

a) Parallels the x - axis

b) Parallels the y - axis

c) Passes through the origin point

d) Intersects the two coordinates axis

6. The third proportional for 9 : 12 , ………………… and 4 is …………………

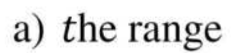
a) 6

b) 3

c) 2

d) 1

7. The simplest and easiest dispersion measure is …………………



b) the arithmetic mean

c) the median

d) the mode

8. If : 3 a = $\frac{5}{6}$ b , then $\frac{a}{b}$ = …………………

a) $\frac{18}{5}$

b) $\frac{15}{6}$

c) $\frac{6}{15}$

d) $\frac{5}{18}$

9. If (3 , 5) ∈ {3 , 6} x {x , 8} , then x = …………………

a) 8

b) 6

c) 3

d) 5

10. If x ={5 , 6 , 7} , then n(x^2) = ...

a) 3

b) 6

c) 9

d) 12

11. If the point (x , 7) lies on the y- axis , then 5 x + 1 = ...

a) 0

b) 1

c) 5

d) 6

12. The set of images of the elements of the domain of the function is called ....

a) the rule

b) the domain

c) the range

d) the codomain

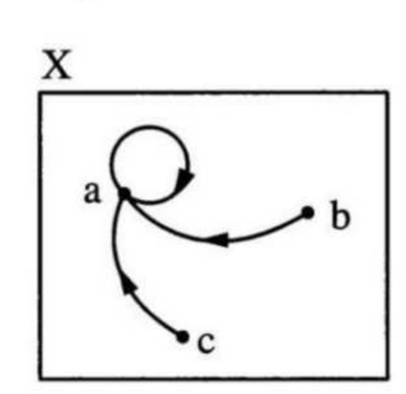
13. The opposite figure represents a function on x , then its range is ....

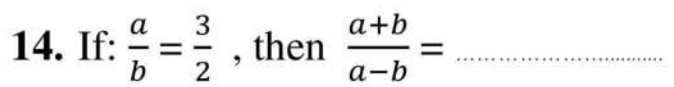
a) {a}

b) {a , b , c}

c) {a , b}

d) {b , c}



14. If: $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b} =$ ....

a) $\frac{4}{5}$

b) $\frac{3}{2}$

c) 2

d) 5

15. If (3 , 5) ∈ {3 , 6} x {n , 8} , then n = ...

a) 8

b) 6

c) 5

d) 3

16. The positive middle proportion between 2 and 8 equals ....

a) 6

b) 4

c) −4

d) 16

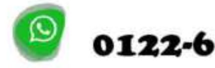
17. The function f where f(x) = x (x - 4) + 1 is a polynomial of the .... degree

a) first

b) second

c) third

d) fourth




18. The fourth proportion for the numbers: 4, 12, 16 is ……………………

a) 20

b) 24

c) 48

d) 64

19. If the point (x-2 , 1) where x ∈ z lies on the first quadrant , then x = …………………

a) 3

b) 1

c) -4

d) zero

20. if : $\frac{a+2b}{a-b} = \frac{3}{2}$, then $\frac{b}{a}$ = ……………………

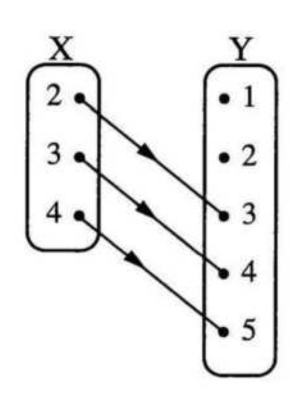
a) $\frac{1}{7}$

b) 8

c) $-\frac{1}{8}$

d) -8

21. The opposite arrow diagram represents a function from x to y, Then the range = ……………………



a) {2 , 3 , 4}

b) {2 , 3 , 5}

c) {3 , 4 , 5}

d) Y

22. The middle proportion between the two numbers 9 and 25 is ……………………

a) 6

b) 15

c) -15

d) $\pm$ 15

23. If the curve of the function F where f(X) $= x^2$ – a , passes through the point (1 , 0) , then a = ……………………

a) ± 1

b) -1

c) 1

d) Zero

24. If : : $\frac{3}{a} = \frac{7}{b} = \frac{k}{b-a} =$, then k = ……………………

a) 3

b) 10

c) 4

d) 7

25. The fourth proportional for the quantities : 3 , 6 , 6 is ……………………

a) 3

b) 6

c) 9

d) 12

26. The ratio between the area of a square shaped region of side length L to the area of another square region of side length 2 L is ……………………

a) 1 : 2

b) L : 4

c) 1 : 4

d) 4 : 1

27. If : 3 x y = 8 , then ……………………

a) $x \propto y$

b) $y \propto x$

c) $3x \propto 8y$

d) $x \propto \frac{1}{y}$

28. The point (-3 , 4) lies in the ……………………………… quadrant

a) first

b) second

c) third

d) fourth

29. If the function f from the set X to the set Y , then the range of ⊂ ……………

a) x

b) y

c) $X\ x\ y$

d) R

2- Complete each of the following.

1. From the methods of collecting data are ………………… , …………………
2. If : (x + 5 , 8) = (1 , 6 y + x) , then y = …………………
3. If : $\frac{a}{b} = \frac{c}{d}$, then $\frac{\text{………}+3c}{5b+\text{………}} = \frac{a}{b}$
4. If : y ∝ x and y = 6 where x = 4 , then $\frac{y}{x}$ = …………………
5. If : n (x) = 5 , n (x y) = 15 , then n (y) = …………………
6. If Ahmed answered 60 % of the questions of a test with true answers and the number of questions which were answered incorrectly are 10 questions , then the number of all questions of the test is …………………
7. The range of the set of values 8 , 5 , 10 , 6 , 14 is ……………………….
8. If the number 6 is the positive mean proportion of the two numbers 2 and a , then a = ……………………………….
9. The point (5 , -3) lies in the ……………………………. quadrant
10. If : x = {2 , 3} , then x^2 = …………………………….
11. If: $\frac{x}{5} = \frac{y}{4} = \frac{x+y}{k}$, then k = ………………………

If: x = {(2 , 6) , (2 , 9) , (3 , 6) , (3 , 9) , (5 , 6) , (5 , 9)}

Find:

1. x ....

2. y ....

3. y^2 ....

3- If: a , b , c and d are proportional quantities, prove that.

$$\frac{a}{b-a} = \frac{c}{d-c}$$

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4- If: x = {1 , 2 , 3} , y = { 2 , 3 , 4 , 5 , 6} and R is a relation from x to y where "aRb" means " $a = \frac{1}{2} b$ " for each a ∈ x and b ∈ y .

Write R and represent it by an arrow diagram, is the relation is a function? Why?

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5- Draw the curve of the function f: $f(X) = x^2 - 4x + 3$ on the interval [0 , 4] and from the graph , find:

a. The coordinates of the vertex point of the curve

b. The maximum or minimum value

c. The equation of the axis of symmetry

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6- If a , b , c , and d are in a continued proportional , prove that.

$$\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$$

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7- If: $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$, prove that: $\frac{2x-y+5z}{3y-x} = 3$

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8- The opposite arrow diagram represents a relation on x, write R and represent it by a Cartesian diagram, is the relation a function? Why?

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X

7 6 8 11 12

9- The ratio between two numbers is 3 : 4 and the difference between them is 25 , find their product

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10- If : x = { k , 8 , 6 , 10} and y = { 3 , 5 , 4 , 7} and R is a relation from x to y where "a R b" means " $b = \frac{a}{2}$ " for each a ∈ x and b ∈ y **Find** the value of k which makes R a function from x to y Represent the function by a Cartesian diagram.

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11- If b is the middle proportion between a and c , prove that : $\frac{a^2+b^2}{b^2+c^2}=\frac{a}{c}$

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12- If a , b , c and d are proportional quantities , then prove that:

$$\frac{5a+3c}{5b+3d}=\frac{3a-3c}{3b-2d}$$

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13- If: 3 a = 2 b = 5 c , then find a : b : c

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14- Graph the function f where f(X) = $x^2 - 2x - 1$ taking x ∈ [-2 , 4] and from the graph , find.

a. Maximum or minimum value of the function f

b. Equation of the axis of symmetry of the curve of the function

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15- If x = {3 , 4} , y = {4 , 5} , z = {6 , 5} Find.

a) X × (y ∩ z)

b) (x – y) × z

c) (x – y) × (y – z)

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16- Represent the function f: R ⟶ R where f(x) = 2 – x and find the points of intersection of the straight line which represents it with the coordinate axes

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17- If : $\frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$, Prove that: $\frac{x+y}{a} = \frac{y+z}{b}$

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18- If : f(x) = a x + 4 and f (1) = 7 find a, then find the value of f(4) + f (2) – f (5)

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19- If : x = {0 , 1 , 4 , 7} , y = {1 , 3 , 5 , 6} and R is a relation from x to y where "a R b" means " a + b < 8 " for each a ∈ x and b ∈ y Write R and represents it by an arrow diagram , is R a function ? why?

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20- If b is the middle proportion between a and c, Then prove that.

$$\frac{3c^2-5\,b^2}{3b^2-5a^2}=\frac{c^2}{b^2}$$

21- Represent graphically the function f where f(X) = $x^2 - 4$ x where x ∈ R taking x ∈ [-1 , 5] and from the graph , deduce the equation of the axis of symmetry

22- If: $\frac{a}{b} = \frac{1}{3}$ and $\frac{c}{d} = \frac{7}{2}$, find the ratio. $\frac{2\,a\,c+b\,d}{b\,c-3\,a\,d}$

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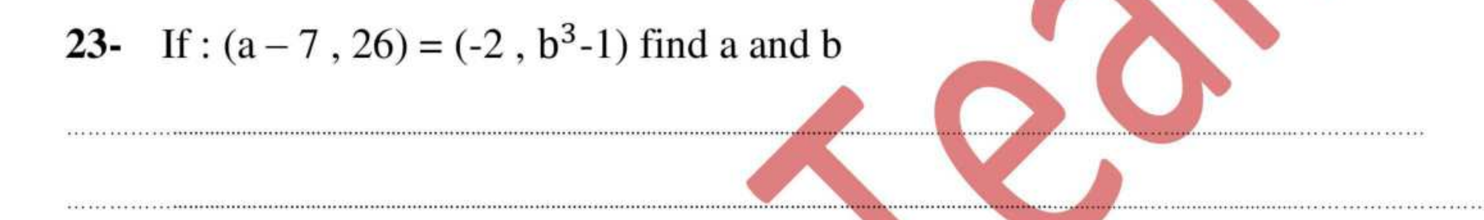
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23- If : (a – 7 , 26) = (-2 , b^3-1) find a and b



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24- If : f(x) = $6x^2$ + a , h (X) = a where f and h are polynomial functions and f(2) + h (2) = 20 , then find: f(-1) – h (100)

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25- If: $x = z + 8$, z varies inversely as y and $z = 2$, where $y = 3$, Find : y when $x = 3$

26- If : $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$, then prove that. $\frac{a-b+c}{a+b-c} = \frac{1}{3}$

27- If : x = { 2 , 3 , 4 , 7} and y = { 1 , 2 , 3 , 4 , 7 , 8} and R is a relation from x to the set y such that "a R b" means " a + b is not a prime number " for all $a \in x$ and $b \in y$ Write R and represent it by an arrow diagram

28- If : x = { 2 , 3 , 4 } and y = { 3 , 4 , 5 , 6 , 7 , 8} and f: x y where $f(x) = 9 - x$, Find the images of the elements of x by the function f

29- If y varies as x and $y = \frac{5}{3}$, when $x = \frac{1}{6}$ find the value of x when $y = \frac{3}{4}$

30- Find the number which should be subtracted from each of the numbers 3 , 7 , 19 to be in continued proportion




31- If: $\frac{x+y}{5} = \frac{y+z}{3} = \frac{z+x}{6}$ prove that: $\frac{x-z}{2} = \frac{x+y+z}{7}$

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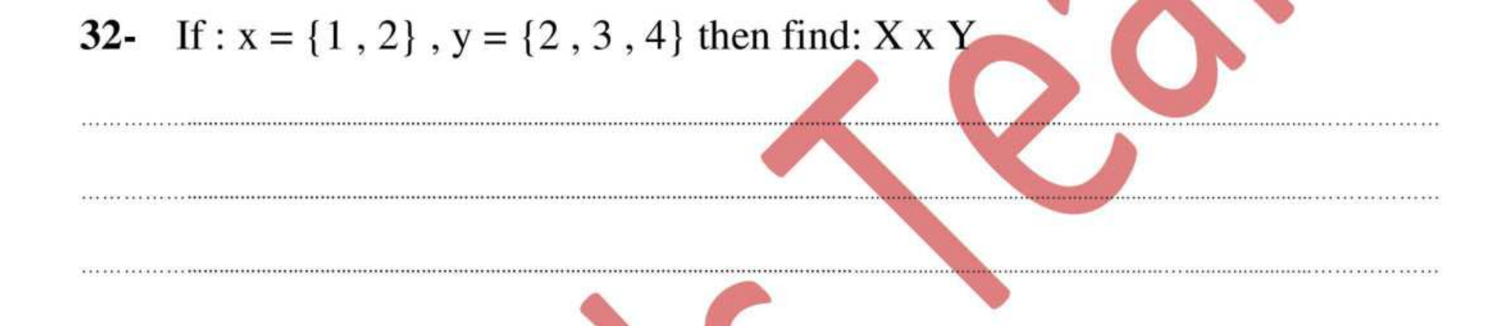
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32- If : x = {1 , 2} , y = {2 , 3 , 4} then find: X x Y

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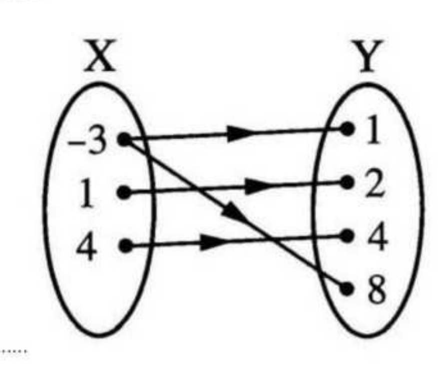
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33- The opposite arrow diagram represents the ration R

From the set x to the set y , where:

x = {-3 , 1 , 4} , y = {1 , 2 , 4 , 8} ,

Write R is R a function? why?



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34- The following table represents the number of children of 100 families in a city:

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate each of the arithmetic mean and the standard deviation

Algebra – Final Revision – Rules

First Algebra

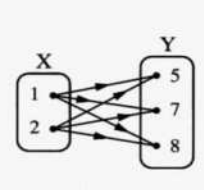
Remember: The Cartesian product of two finite sets and represent it

If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

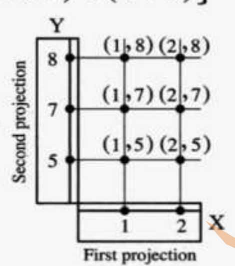
X × Y

is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y

i.e. $X \times Y = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$



The arrow diagram

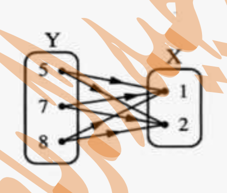


The graphical diagram (The Cartesian diagram)

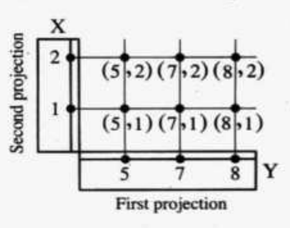
Y × X

is the set of all ordered pairs whose first projection of each of them belongs to Y and the second projection of each of them belongs to X

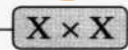
i.e. $Y \times X = \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$



The arrow diagram

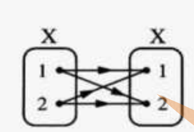


The graphical diagram (The Cartesian diagram)

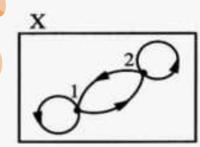


is the set of all ordered pairs whose first projections and second projections belong to X

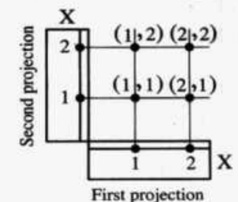
i.e. $X \times X = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$



The arrow diagram



The arrow diagram



The graphical diagram (The Cartesian diagram)

Remark

(1) $X \times Y \neq Y \times X$, where : $X \neq Y$

(2) $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$ where n is the number of elements

(3) $n(X \times X) = n(X^2) = [n(X)]^2$

(4) $X \times \varnothing = \varnothing \times X = \varnothing$

Remember: The relation and its representing

- The relation from the set X to the set Y is a connecting joining some or all the elements of X with some or all the elements of Y
- If R is a relation from the set X to the set Y, then :

1. R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y
2. $R \subset X \times Y$
3. The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically)

- If R is a relation from X to X , then R is a relation on X and $R \subset X \times X$

Remember **The function**

- **A relation from X to Y is said to be a function if :**

1. Each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.
2. Each element of the set X has one and only one arrow going out of it to one element of Y in the arrow diagram which represents the relation.
3. Each vertical line has one and only one point lying on it of the points which represent the relation , in the Cartesian diagram which represents the relation.

- If f is a function from the set X to the set Y is written as $f : X \longrightarrow Y$, then :

1. X is called the domain of the function f
2. Y is called the codomain of the function f
3. The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y

Remember **The polynomial functions**

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (The index) of the variable x in any of its terms is a natural number with noticing that : the degree of the function is the highest power of the variable x

For example :

- The function $f : f(x) = 3$ is a polynomial function of zero degree.
- The function $f : f(x) = 2x + 1$ is a polynomial function of the first degree.
- The function $f : f(x) = x^3 - 5x^2 + 1$ is a polynomial function of the third degree.

While :

The function f: $f(x) = \frac{1}{x^2} + x^2$ is not a polynomial function because : $\frac{1}{x^2} = x^{-2}$

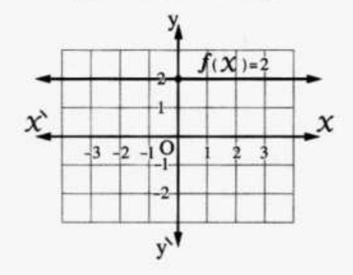
i.e. the index of the symbol x is not a natural number.

Remember — The graphical representation of polynomial function

1 The constant function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is represented by a straight line parallel to X-axis and intersects y-axis at the point (0 , b)

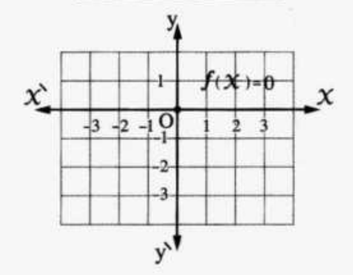
$f : f(X) = 2$



The straight line above X-axis and passes through the point (0 , 2)

(is of zero degree)

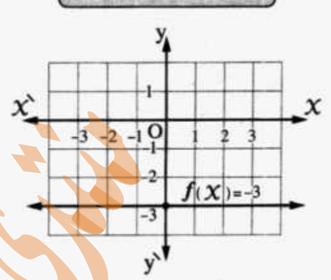
$f : f(X) = 0$



The straight line is coincident with X-axis and passes through the point (0 , 0)

(has not degree)

$f : f(X) = -3$



The straight line below X-axis and passes through the point (0 , – 3)

(is of zero degree)

Remember — The ratio and its properties

- The ratio between the two real numbers a and b is written as a : b or $\frac{a}{b}$ and a is called the antecedent of the ratio , b is called the consequent and a , b are called the two terms of the ratio.
- If the ratio between two numbers is a : b , then : The first number = am , The second number = bm , $m \neq 0$

Example

Two numbers , their sum is 28 and the ratio between them is 3 : 4 what are the two numbers ?

Solution

Let the two numbers be 3 m , 4 m $\therefore 3m + 4m = 28$ $\therefore 7m = 28$ $\therefore m = \frac{28}{7} = 4$

$\therefore$ The two numbers are : 3×4 and 4×4 *i.e.* 12 and 16

Remember — The proportion

- The proportion is the equality of two ratios or more.
- If $\frac{a}{b} = \frac{c}{d}$, then a , b , c and d are proportional quantities.
- If a , b , c and d are proportional quantities , then $\frac{a}{b} = \frac{c}{d}$

Remember: The properties of the proportion

Property 1

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$

i.e. the product of the extremes = the product of the means.

Example Find the fourth proportional of the quantities : 3 , 4 and 27

Solution

Let the fourth proportional be x ∴ The quantities : 3 , 4 , 27 and x are proportional

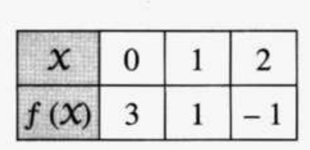
∴ $\frac{3}{4} = \frac{27}{x}$ ∴ $3 \times x = 4 \times 27$ ∴ $x = \frac{4 \times 27}{3} = 36$ ∴ The fourth proportional = 36

2 The linear function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where : $f(x) = a x + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called linear function (function of first degree) and is represented by a straight line intersects y-axis at (0 , b) and x-axis at $\left(\frac{-b}{a}, 0\right)$

$f : f(x) = 2x - 3$

x	0	1	2
$f(x)$	−3	−1	1

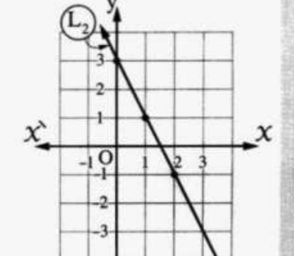


The straight line L_1 intersects :

• x-axis at $\left(1\frac{1}{2}, 0\right)$ • y-axis at (0 , −3)

$f : f(x) = 3 - 2x$

x	0	1	2
$f(x)$	3	1	−1



The straight line L_2 intersects :

• x-axis at $\left(1\frac{1}{2}, 0\right)$ • y-axis at (0 , 3)

3 The quadratic function

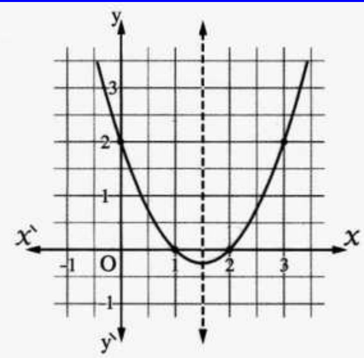
The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where : $f(x) = a x^2 + b x + c$, a , b and c $\in \mathbb{R}$, $a \neq 0$ is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$f : f(x) = x^2 - 3x + 2$, $x \in [0, 3]$

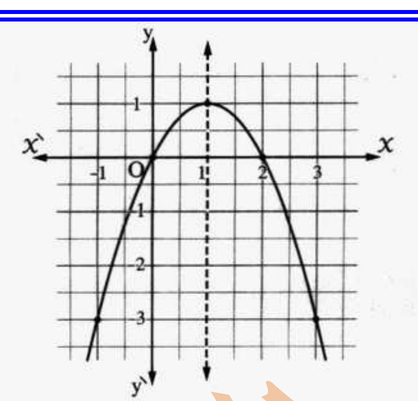
x	0	1	2	3
$f(x)$	2	0	0	2

$f : f(x) = 2x - x^2$, $x \in [-1, 3]$

x	−1	0	1	2	3
$f(x)$	−3	0	1	0	−3



- The vertex of the curve $= \left(\frac{3}{2} , -\frac{1}{4}\right)$
- The minimum value of the function $= -\frac{1}{4}$
- The equation of line of symmetry : $x = \frac{3}{2}$



- The vertex of the curve $= (1 , 1)$
- The maximum value of the function $= 1$
- The equation of line of symmetry : $x = 1$

Example

If $\frac{x + 3y}{2x - y} = \frac{4}{3}$, then find the ratio $x : y$

Solution

$\because \frac{x + 3y}{2x - y} = \frac{4}{3}$ $\quad \therefore 3(x + 3y) = 4(2x - y)$ $\quad \therefore 3x + 9y = 8x - 4y$

$\therefore 13y = 5x$ $\quad \therefore x : y = 13 : 5$

Remember — The continued proportion

- The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

a is called the **first proportion** , **c** is called **the third proportion** and

b is called the **middle proportion (Proportional mean)**

- $\because \frac{a}{b} = \frac{b}{c}$ $\quad \therefore b^2 = ac$ $\quad \therefore b = \pm\sqrt{ac}$

i.e.

The middle proportion between two quantities $= \pm\sqrt{\text{the product of the two quantities}}$

Notice that :

The two quantities a and c should be either positive together or negative together.

- If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\begin{cases} c = dm \\ b = dm^2 \\ a = dm^3 \end{cases}$

Remember The direct variation and inverse variation

Direct variation

- If y varies directly as x

and is written as $y \propto x$, then :

1. $y = m x \left(i.e. \frac{y}{x} = m\right)$

where m is a constant $\neq 0$

2. $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

3. The relation between x and y is represented graphically by a straight line passing through the origin point

- To prove that $y \propto x$,

we prove that : $y = m x$

where m is a constant $\neq 0$

For example :

If : $y = 5x$, then : $y \propto x$

Inverse variation

- If y varies inversely as x

and is written as $y \propto \frac{1}{x}$, then :

1. $y = \frac{m}{x} \left(i.e. xy = m\right)$

where m is a constant $\neq 0$

2. $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

3. The relation between x and y is not linear relation.

- To prove that $y \propto \frac{1}{x}$,

we prove that : $x y = m$

where m is a constant $\neq 0$

For example :

If : $y = \frac{7}{x}$, then $x y = 7$, and then $y \propto \frac{1}{x}$

Remember The resources of collecting data

Primary resources (field resources)

- These are the resources from which we get data directly.

Example :

* Questionnaires and survey.
* Observing and measuring.
* The personal interview.

Secondary resources (historical resources)

- These are the resources from which we get data that previously collected.

Example :

* Central agency for public mobilization and statistics.
* Mass-media.
* Internet.

Remember The methods of collecting data

Method of mass population | **Method of sample**

Remember The concept of the sample and the methods of collecting it

The sample :
It is a small part from a large society that looks like the society and represents it well.

The methods of collecting the sample and its types

- **Biased selection**
- **Randomly selection**

The number of individuals of the layer in the sample

$= \dfrac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times$ the number of individuals of the sample

«approximated the result of the nearest unit»

Remember — The dispersion and its measurements

The dispersion :

It is a measure that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion)

It is the difference between the greatest value and the smallest value in the set.

i.e. The range = the greatest value – the smallest value

2 Standard deviation

The standard deviation of a set of values

The standard deviation $\sigma = \sqrt{\dfrac{\sum (x - \overline{x})^2}{n}}$

Where :

x denotes a value of the values ,

$\overline{x}$ denotes the mean of the values ,

n denotes the number of values ,

$\sum$ denotes the summation operation.

The standard deviation of a frequency distribution

The standard deviation $\sigma = \sqrt{\dfrac{\sum (x - \overline{x})^2 k}{\sum k}}$

Where :

x represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

$\sum k$ is the sum of frequences

and $\overline{x}$ (the mean) $= \dfrac{\sum (x \times k)}{\sum k}$

Remember that

The centre of the set $= \dfrac{\text{lower limit} + \text{upper limit}}{2}$

[A] Choose the correct Answer :

1	If $X = \{5\}$ and $Y = \{3\}$, then $n(X \times Y) =$ …………… (a) 15 (b) 5 (c) 3 (d) 1	
2	If $X = \{7\}$, $Y = \{5\}$, then $n(X \times Y) =$ …………… (a) 1 (b) 2 (c) zero (d) 35	
3	If $n(X) = 2$, $Y = \{1 , 2\}$, then $n(X \times Y) =$ …………… (a) 4 (b) 3 (c) 5 (d) 6	
4	If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ …………… (a) 15 (b) 36 (c) 4 (d) 9	
5	If $n(X^2) = 4$, $n(X \times Y) = 12$, then $n(Y) =$ …………… (a) 3 (b) 6 (c) 4 (d) 12	
6	If $X = \{2 , 3 , 4\}$, then $n(X^2) =$ …………… (a) 3 (b) 6 (c) 9 (d) 12	
7	If $X = \{2\}$, then $X^2 = \{$ …………… $\}$ (a) (2 , 2) (b) (2 , 0) (c) (0 , 2) (d) (2 , – 2)	
8	If $(2 , 9) \in \{2 , 8\} \times \{x , 4\}$, then $x =$ …………… (a) 8 (b) 6 (c) 9 (d) 2	
9	If $\{2\} \times \{x , y\} = \{(2 , 4) , (2 , 3)\}$, then $x - y =$ …………… (a) 1 (b) – 1 (c) ± 1 (d) zero	
10	The set of images elements of the domain of the function is called …………… (a) the rule. (b) the domain. (c) the range. (d) the codomain.	
11	The point (4 , – 5) lies in the …………… quadrant. (a) first (b) second (c) third (d) fourth	

No.	Question
12	If the point $(X - 4 , 2 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant , then X equals .... (a) 2 (b) 3 (c) 4 (d) 6
13	If the point $(X - 5 , 7 - X)$ lies in the second quadrant , then $X =$ .... (a) 5 (b) 3 (c) 7 (d) 9
14	If the point $(X , 2)$ lies on y-axis , then $X + 3 =$ .... (a) 3 (b) zero (c) 2 (d) 5
15	if $(X - 2 , 13) = (7 , y + 5)$, then $\sqrt{X + 2y} =$ .... (a) 2 (b) 5 (c) 7 (d) 13
16	If $(X - 1 , 11) = (8 , y + 3)$, then $\sqrt{X + 2y} =$ .... (a) 7 (b) 5 (c) 4 (d) 3
17	If the point (a , 3) lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 4X - 5$, then a equals .... (a) 2 (b) 4 (c) 5 (d) – 1
18	If the point (a , 2) lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X - 1$, then a = ... (a) 2 (b) 3 (c) 1 (d) – 1
19	The function f where $f(X) = X^4 - 2X^3 + 7$ is polynomial of degree .... (a) first. (b) second. (c) third. (d) fourth.
20	The function f where $f(X) = X(X - 4) + 1$ is a polynomial of the .... degree. (a) first (b) second (c) third (d) fourth
21	If $f : \mathbb{R} \longrightarrow \mathbb{R}$, then the function f where $f(X) = X^2 - (X^2 - 3X)$ is of the .... degree. (a) first (b) second (c) third (d) fourth
22	$f(X) = X(3X + 2)^2$ is function of .... degree. (a) third (b) second (c) first (d) otherwise

23	If (2 , b) satisfies the function f where $f(\mathcal{X}) = 3\mathcal{X} - 6$, then b = ... (a) zero (b) 7 (c) 9 (d) 2
24	If $f(\mathcal{X}) = k\mathcal{X} + 8$, $f(2) = \text{zero}$, then k = ... (a) 8 (b) 6 (c) 4 (d) – 4
25	The function f where $f(\mathcal{X}) = 5\mathcal{X}$ is represented graphically by a straight line passes through the point .... (a) (5 , 5) (b) (0 , 0) (c) (0 , 5) (d) (5 , 0)
26	If 3a = – 4b , then $\frac{a}{b}$ = ... (a) $-\frac{4}{3}$ (b) $-\frac{3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
27	If 6 a – 5 b = 0 , then $\frac{a}{b}$ = ... (a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
28	If $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b}$ equals .... (a) $\frac{5}{3}$ (b) 1 (c) 3 (d) 15
29	If $3a = \frac{5}{6}b$, then $\frac{a}{b}$ = ... (a) $\frac{18}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ (d) $\frac{5}{18}$
30	If $\frac{\mathcal{X}}{2} = \frac{y}{3}$, then $\frac{3\mathcal{X}}{2y}$ = ... (a) zero (b) 1 (c) 2 (d) 3
31	If $\frac{a}{3} = \frac{b}{2} = \frac{2a+b}{\mathcal{X}}$, then $\mathcal{X}$ = ... (a) 8 (b) 4 (c) 3 (d) 1
32	If $\frac{\mathcal{X}}{8} = \frac{y}{7} = \frac{\mathcal{X}+y}{3k}$, then k = ... (a) 15 (b) 8 (c) 5 (d) 7
33	The equality of two ratios or more is called the .... (a) function. (b) direct variation. (c) inverse variation. (d) proportion.

34	The first proportional for the numbers : 7 , 10 , 14 is .... (a) 5 (b) 7 (c) 10 (d) $\frac{7}{10}$
35	If 4 , x , 12 , 18 are proportional , then x = ... (a) 2 (b) 3 (c) 6 (d) 54
36	The third proportional of the numbers : 4 , 3 , ... , 6 is .... (a) 2 (b) 4 (c) 5 (d) 8
37	The fourth proportional of the quantities : 9 , 21 , 15 , ... is .... (a) 14 (b) 28 (c) 35 (d) 42
38	The third proportion of the two numbers 3 and 6 is .... (a) $\frac{1}{2}$ (b) 2 (c) 6 (d) 12
39	The positive middle proportion between 3 and 27 is .... (a) 3 (b) 4 (c) 8 (d) 9
40	If 2 , 6 , x + 15 are proportional , then x = ... (a) 1 (b) 2 (c) 3 (d) 4
41	If 3 , x and $\frac{1}{y}$ are in continued proportion , then y $\propto$ .... (a) x (b) $\frac{1}{x}$ (c) x^2 (d) $\frac{1}{x^2}$
42	If 1 , x , 4 in continued proportion , then x = ... (a) ± 1 (b) ± 2 (c) ± 4 (d) ± 3
43	) If y varies inversely with x and y = 4 when x = 3 , then y = ... when x = 2 (a) 4 (b) 3 (c) 6 (d) 12
44	If y = 4 x , then .... (a) $y \propto \frac{1}{x}$ (b) $x \propto \frac{1}{y}$ (c) $y \propto x$ (d) otherwise.
45	If y = 2 x , then y $\propto$ .... (a) $\frac{1}{x}$ (b) x^2 (c) $\frac{1}{x^2}$ (d) x

46 If $\frac{x}{y} = 1$, then $y \propto$ ....

(a) $x - 1$ (b) $x + 1$ (c) $\frac{1}{x}$ (d) x

47 If $x y = 2$, then ....

(a) $x \propto \frac{1}{y}$ (b) $x \propto y$ (c) $x \propto 2y$ (d) $x = 2y$

48 If $x y^2 = a$ where is a constant $\neq$ zero , then x varies inversely with ....

(a) $\frac{1}{y^2}$ (b) $\frac{1}{y}$ (c) y (d) y^2

49 If $2x - 3y = 2y - 5x$, then $x \propto$ ....

(a) y^2 (b) $\frac{1}{y^2}$ (c) $\frac{1}{y}$ (d) y

50 The relation represents the direct variation between the two variables x and y which is ....

(a) $x y = 5$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$

51 If $y^2 + 4x^2 = 4xy$, then ....

(a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

52 If $4x^2 - 12xy + 9y^2 = 0$, then $y \propto$ ....

(a) x (b) x^2 (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

53 If $x^2 - 4xy + 4y^2 = 0$, then ....

(a) $x \propto y$ (b) $x \propto \frac{1}{y}$ (c) $x \propto y^2$ (d) $x \propto \frac{1}{y^2}$

54 One of the measurements of dispersion is the ....

(a) arithmetic mean. (b) median. (c) range. (d) mode.

55 The simplest and easiest dispersion measure is the ....

(a) range. (b) arithmetic. (c) median. (d) mode.

56 The range of the set of the values : 7 , 3 , 6 , 9 and 5 equals ....

(a) 3 (b) 4 (c) 6 (d) 12

57 The range of the set of the values : 3 , 4 , 6 , 9 , and 12 equals ....

(a) 15 (b) 9 (c) 5 (d) 3

58 The range of the set of the values : 5 , 14 , 21 , 4 , 16 , 12 is ....

(a) 17 (b) 15 (c) 13 (d) 11

59 The range of the set of the values : 5 , 13 , 4 , 19 and 16 is ....

(a) 19 (b) 16 (c) 14 (d) 15

60 The range of the set of values : 23 , 22 , 15 , 18 , 17 is ....

(a) 8 (b) 18 (c) 19 (d) 23

61 The difference between the largest and smallest value for a set of values is the ....

(a) arithmetic mean. (b) range. (c) median. (d) mode.

62 The difference between the maximum and minimum value is called the ....

(a) range. (b) mean. (c) median. (d) standard deviation.

63 The difference between the greatest value and the smallest value is ....

(a) the range. (b) the mean. (c) the median. (d) the standard defiation.

64 The set which has the greatest dispersion in the following sets is ....

(a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43

(c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27

65 The arithmetic mean of the set of the values : 6 , 2 , 8 and 4 = ...

(a) 5 (b) 6 (c) 10 (d) 20

66 The mean of the values : 2 , 3 , 7 , 8 , 10 = ...

(a) 3 (b) 5 (c) 2 (d) 6

67 The arithmetic mean of the set of the values : 7 , 3 , 6 , 9 and 5 equals ....

(a) 3 (b) 4 (c) 6 (d) 12

68 The arithmetic mean of the set of the values : 7 , 6 , 5 , 13 , 4 is ....

(a) 9 (b) 15 (c) 6 (d) 7

69	The arithmetic mean of the values : 12 , 24 , 26 , 38 and 20 is .... (a) 12 (b) 24 (c) 26 (d) 38
70	The positive square root of the average of the squares of deviations of the values from their arithmetic mean is called .... (a) the range. (b) the arithmetic mean. (c) the median. (d) the standard deviation.
71	If all the individuals are equal in values then .... (a) $x - \overline{x} >$ zero (b) $x - \overline{x} <$ zero (c) $\sigma =$ zero (d) $\overline{x} =$ zero
72	If $\Sigma (x - \overline{x})^2 = 36$ for a set of values whose number is 9 , then $\sigma =$ .... (a) 2 (b) 4 (c) 18 (d) 27
73	If $\Sigma (x - \overline{x})^2 = 28$ for a set of values whose number is 7 , then $\sigma =$ .... (a) 2 (b) 4 (c) 7 (d) 14
74	If the standard deviation of the set of values = 2 and number of these values = 6 , then $\Sigma (x - \overline{x})^2 =$ .... (a) 12 (b) 18 (c) 24 (d) 36

Choose the correct Answers

Sn.	Answer	Sn.	Answer	Sn.	Answer	Sn.	Answer
1	D	21	A	41	C	61	B
2	A	22	A	42	B	62	A
3	A	23	A	43	C	63	A
4	C	24	D	44	C	64	D
5	B	25	B	45	D	65	A
6	C	26	A	46	D	66	D
7	A	27	B	47	A	67	C
8	C	28	B	48	D	68	D
9	C	29	D	49	D	69	B
10	C	30	B	50	D	70	D
11	D	31	A	51	A	71	C
12	B	32	C	52	A	72	A
13	B	33	D	53	A	73	A
14	A	34	A	54	C	74	C
15	B	35	C	55	A	75	
16	B	36	D	56	C	76	
17	A	37	C	57	B	77	
18	C	38	D	58	A	78	
19	D	39	D	59	D	79	
20	B	40	C	60	A	80	

[B] Essay Problems : -

1	If $(2x, 4) = (8, y+1)$, then find the value of : $\sqrt{x^2 + y^2}$ 2016 Exam (13) Question (5) (a)
2	Find : a , b if $(a-7, 26) = (-2, b^3 - 1)$ 2016 Exam (2) Question (5) (a)
3	If $(x-1, 11) = (8, y+3)$, then find : $\sqrt{x + 2y}$ 2016 Exam (3) Question (2) (a)
4	If $(x^2, 27) = (1, y^3)$ and the point (x, y) lies in the second quadrant , find the value of : $\sqrt{y - x}$ 2016 Exam (9) Question (4) (a)
5	If $X = \{1, 2\}$, $Y = \{2, 3, 4\}$, then find : $X \times Y$ 2016 Model (2) Question (4) (b)
6	If $X = \{2, 3\}$, then find : X^2 2016 Exam (4) Question (4) (b)
7	If $X = \{2, 5\}$, $Y = \{1, 3, 7\}$, then find : (1) $X \times Y$ (2) $n(Y^2)$ 2016 Exam (2) Question (3) (b)
8	If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ Find : (1) X , Y (2) $Y \times X$ 2016 Exam (8) Question (2) (a)
9	If $X \times Y = \{(2, 3), (2, 2), (2, 4)\}$ Find each of the following : (1) X , Y (2) $X \times (X \cap Y)$ 2016 Exam (9) Question (2) (a)
10	If $X \times Y = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$ Then find : (1) $X \cup Y$ (2) Y^2 2016 Exam (12) Question (4) (b)

No.	Question
11	If $X = \{2, 15\}$, $Y = \{4, 1\}$ and $Z = \{15\}$ **Find :** (1) $Y \times Z$ (2) $n(X^2)$ (3) $(X \cap Z) \times Y$ **2016 Exam (1) Question (2) (a)**
12	If $X = \{1, 2\}$, $Y = \{2, 5\}$, $Z = \{4, 5\}$, then : **Find :** $(X \cap Y) \times (Y \cup Z)$ **2016 Exam (15) Question (2) (a)**
13	If $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :** (1) $X \times (Y \cap Z)$ (2) $(X - Y) \times Z$ **2016 Exam (3) Question (5) (a)**
14	If $X = \{-5, -3, -1\}$, $Y = \{1, 2, 3, 5\}$ and R is a relation from X to Y where "a R b" means "a is additive inverse of b" for all $a \in X$, $b \in Y$ (1) Write R and represent it by an arrow diagram. (2) Show that R is a function from X to Y and find its range. **2016 Exam (12) Question (3) (a)**
15	If R is a relation on $\mathbb{N}$ (set of natural numbers), where "a R b" means "$a \times b = 18$" for all $a \in \mathbb{N}$, $b \in \mathbb{N}$. Write R and represent it by an arrow diagram. **2016 Model (3) Question (5) (a)**
16	If $X = \{1, 2, 5, 7\}$, $Y = \{2, 3, 7, 8\}$ and R is a relation from X to Y where "a R b" means "$a + b = 9$" for all $a \in X$, $b \in Y$ (1) Find the relation R (2) Represent the relation R by an arrow diagram. (3) Is R a function ? Why ? **2016 Exam (8) Question (3) (a)**
17	If $X = \{2, 3, 4\}$, $Y = \{3, 4, 5, 6, 7, 8\}$ and $f : X \longrightarrow Y$ where $f(x) = 9 - x$ Find the images of the elements of X by the function f **2016 Model (1) Question (5) (a)**
18	If $X = \{2, 3, 4\}$, $Y = \{3, 4, 5, 6, 7, 8\}$ and R is a relation from x to y where "a R b" means that "$a = \frac{1}{2}$ b" for all $a \in X$, $b \in Y$ (1) Write R (2) Represent R by arrow diagram. (3) Is R a function ? Why ? **2016 Exam (9) Question (3) (b)**

19	If $X = \{2, 6, 8\}$, $Y = \{3, 4, 8, 9\}$, R is a relation from X to Y, "a R b" means "a + b = an odd number" for every $a \in X$, $b \in Y$ Write R as a set of orderd pairs. Is R a function ? Why ? **2016 Exam (11) Question (3) (b)**
20	If $X = \{2, 3, 4, 7\}$, $Y = \{1, 2, 3, 4, 7, 8\}$ and R is a relation from X to Y, where "a R b" means "a – b is an odd number" for all $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram. **2016 Model (5) Question (5) (a)**
21	If $X = \{2, 3, 4, 7\}$, $Y = \{1, 2, 3, 4, 7, 8\}$, R is a relation from the set X to the set Y such that "aRb" means "a + b is not a prime number" for all $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram. **2016 Model (1) Question (4) (b)**
22	If $X = \{1, 2, 3\}$, $Y = \{1, 4, 9, 10\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt{b}$" for each $a \in X$, $b \in Y$ (1) Write the relation R and represent it by an arrow diagram. (2) Show that R is a function and write its range. **2016 Exam (5) Question (2) (b)**
23	If function $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$ (1) Write each of domain and range of f (2) Write the rule of the function f **2016 Exam (10) Question (3) (a)**
24	The opposite figure represents the arrow diagram of the given relation R on the set $X = \{1, 2, 3, 4, 5\}$ Write the relation R and represent it by a Cartesian diagram. X 1 2 3 4 5 **2016 Exam (15) Question (2) (b)**
25	The opposite arrow diagram represents the relation R from the set X to the set Y, where : $X = \{-3, 1, 4\}$, $Y = \{1, 2, 4, 8\}$, Write R. Is R a function ? why ? X Y -3 1 4 1 2 4 8 **2016 Model (2) Question (5)**

26	Represent graphically the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3X - 2$ **2016 Exam (15) Question (5) (a)**
27	If the point (a , 5) lies on the straight line which is represented by the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X - 4$, then find the value of a **2016 Exam (11) Question (5) (b)**
28	Represent graphically the linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X + 2$, from the graph find the surface area of the triangle bounded by the straight line which represents the function and the two coordinate axes. **2016 Exam (1) Question (5) (b)**
29	If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is represented by a straight line cuts y-axis at (b – 2 , 5) where $f(X) = 2X - a$ **Find the value of :** 3 a + 2 b **2016 Exam (6) Question (3) (b)**
30	If the straight line that represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - a$ cuts y-axis at the point (b , 3) **Find the value of :** 2 a + 7 b **2016 Exam (4) Question (3) (a)**
31	If f is a function where $f(X) = 3X + 4$ is represented graphically by a straight line passes through the point (a , – 5) **Find :** (1) $f\left(\frac{2}{3}\right)$ (2) The value of a **2016 Exam (12) Question (2) (a)**
32	**In the opposite figure :** If the function f is represented graphically by a straight line $\overleftrightarrow{AB}$ where $A \in \overleftrightarrow{XX'}$, $B \in \overleftrightarrow{yy'}$, OA = 3 length units , O (0 , 0) and the function r : r $(X) = 6$ is represented graphically by a straight line $\overleftrightarrow{BC}$ (1) **Find :** The rule of the function f (2) **Find the value :** $f(6) + r(1)$ **2016 Exam (5) Question (3) (b)**
33	If : f is a polynomial function where $f(X) = X^2 - X + 3$ **Find :** $f(-2)$, $f(0)$, and $f(\sqrt{3})$ **2016 Exam (14) Question (5) (a)**

34

If $f(X) = X^2 - 3X$, $g(X) = X - 3$

(1) **Find :** $f\left(\sqrt{3}\right) + 3\,g\left(\sqrt{3}\right)$ (2) **Prove that :** $f(3) = g(3) = 0$

2016 Exam (13) Question (4) (a)

35

Draw the curve of the function : $f(X) = X^2 + 1$ on the interval $[-3 , 3]$ and from the graph find :

(1) The coordinate of the vertex of the curve.

(2) The equation of the axis of symmetry. (3) The minimum value.

2016 Exam (8) Question (4) (a)

36

Graph the function f where $f(X) = 4 - X^2$ in the interval $[-3 , 3]$, from the graph **determine :**

(1) The coordinates of the vertex of the curve.

(2) The equation of the axis of symmetry.

(3) The maximum value of the function.

2016 Exam (3) Question (4) (a)

37

Represent graphically the function $f(X) = (X - 3)^2$, taking $X \in [0 , 6]$ from the graph

Deduce : (1) The vertex of the curve.

(2) The maximum or the minimum value of the function.

(3) The equation of the axis of symmetry.

2016 Exam (6) Question (5) (a)

38

If the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$

where $f(X) = m - X^2$ cuts X-axis at the point $(-2 , b)$ **Find the value :** $m^b + 2m$

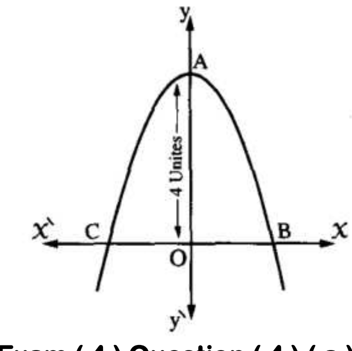
2016 Exam (5) Question (2) (a)

39

The opposite figure represents the curve of the function f where $f(X) = m - X^2$, if OA = 4 units

Find : (1) The value of m

(2) The coordinates of B and C



If $X = \{2 , 3\}$, **then find :** X^2

2016 Exam (4) Question (4) (a)

40

If $\frac{X}{y} = \frac{2}{5}$, what is the value of the expression : $\frac{2X + y}{X + 4y}$?

2016 Model (5) Question (3) (a)

No.	Question
41	If x , y , 2 , 3 are proportional quantities. **Then find :** The value of the ratio $\frac{x+y}{5x-y}$ 2016 Exam (12) Question (4) (a)
42	If a , b , c , d are proportional quantities **Prove that :** $\frac{a}{b-a}=\frac{c}{d-c}$ 2016 Exam (1) Question (3) (a)
43	If $5a = 2b$ **find the value of :** $\frac{3a+2b}{4a+5b}$ 2016 Exam (9) Question (3) (a)
44	If $\frac{a}{4}=\frac{b}{5}=\frac{c}{3}$, **then prove that :** $\frac{a-b+c}{a+b-c}=\frac{1}{3}$ 2016 Exam (6) Question (2) (a)
45	If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$, **then prove that :** $3x^2+3y^2+z^2=(2x+y)^2$ 2016 Exam (3) Question (2) (b)
46	If $a : b : c = 5 : 7 : 3$ and $a + b = 24$, **then find the value of :** a , b and c 2016 Exam (8) Question (3) (b)
47	If $\frac{x}{2}=\frac{y}{3}=\frac{y-x}{5k}$, **find the value of :** k 2016 Exam (7) Question (3) (b)
48	If $\frac{a}{b-a}=\frac{c}{d-c}$, **then prove that :** a , b , c and d are proportional. 2016 Exam (5) Question (3) (a)
49	If $\frac{x+y}{5}=\frac{y+z}{3}=\frac{z+x}{6}$ **Prove that :** $\frac{x-z}{2}=\frac{x+y+z}{7}$ 2016 Model (2) Question (4) (a)
50	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 , find the two numbers. 2016 Exam (10) Question (2) (a)
51	Find the number that if is added to the numbers 7 , 9 , 12 and 15 they become proportional. 2016 Exam (14) Question (3) (b)

52 Find the number which is subtracted from each of the following numbers to be proportional 16 , 21 , 14 and 18

2016 Exam (4) Question (5) (a)

53 If b is a middle proportion between a , c **Prove that :** $\frac{b}{b+c} = \frac{a}{a+b}$

2016 Exam (9) Question (5) (a)

54 If b is the middle proportion between a , c

Prove that : $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$

2016 Exam (7) Question (2) (b)

55 If a , b , c and d are in continued proportion , **prove that :** $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

2016 Model (5) Question (4) (b)

56 If $\frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2}$, **then prove that :** b is the middle proportional between a and c

2016 Model (4) Question (3) (b) 2016 Exam (3) Question (3) (b)

57 Find the number that if we add it to each of the numbers 1 , 7 , 25 , then they become in continued proportion.

2016 Model (1) Question (3) (b)

58 If $y \propto X$ and $y = 21$ when $X = 7$

(1) Find the relation between X and y

(2) Find the value of y when $X = 4$

2016 Exam (14) Question (4) (b)

59 If y varies as X and $y = \frac{5}{3}$, when $X = \frac{1}{6}$ Find the value of X when $y = \frac{3}{4}$

2016 Model (2) Question (3) (a)

60 If $y \propto \frac{1}{X}$ and $y = 5$ when $X = 3$

(1) Find the relation between X and y

(2) Find the value of y when $X = 7.5$

2016 Exam (13) Question (4) (b)

61 If y varies inversely as X and $y = 10$ when $X = 3$

Find the relation between X and y , **then find the value of :** y when $X = 5$

2016 Exam (8) Question (4) (b)

62

If the weight of a body on the earth (w) varies as its weight on the moon (r)

If $w_1 = 182$ kg when $r_1 = 35$ kg , find r_2 when $w_2 = 312$ kg.

2016 Model (4) Question (5) (b)

63

If the velocity (v) of the water running through a pipe varies inversely as the square of the radius of the pipe (r) and v = 5 cm./sec. when r = 3 cm.

Find : v when $r = 15\frac{3}{4}$ cm.

2016 Model (5) Question (5) (b)

64

If $y = 5 + a$ and $a \propto x$ where $a = 6$ when $x = 2$

(1) Find the relation between : y , x

(2) Find the value x when : $y = 8$

2016 Exam (5) Question (4) (a)

65

If $y = a + 7$ and $a \propto \frac{1}{x^2}$, if $a = 3$ when $x = 2$, **then find :**

(1) The relation between y and x

(2) The value of y when $x = \sqrt{3}$

2016 Exam (6) Question (3) (a)

66

From the data of the following table answer the following questions :

(1) Identify the kind of variation between x and y

(2) Find the constant of variation.

(3) Find the value of y when $x = 3$

x	2	4	6
y	6	3	2

2016 Exam (3) Question (4) (b)

67

If $x^2 y^2 - 8xy + 16 = 0$, **then prove that :** $x \propto \frac{1}{y}$

2016 Exam (4) Question (3) (b)

68

If $x^2 + 4y^2 = 4xy$, **then prove that :** y varies directly with x

2016 Exam (13) Question (3) (a)

69

If $a^2 b^4 - 10ab^2 + 25 = 0$

Prove that : a varies inversely with b^2

2016 Exam (2) Question (2) (b)

70 If $a^2 b^2 + \frac{1}{4} = ab$, **then prove that :** a varies inversely as b

2016 Model (4) Question (3) (a)

71 If $4 x^2 + 9 y^2 = 12 x y$, **then prove that :** x varies as y

2016 Model (5) Question (3) (b)

72 If $\frac{21 x - y}{7 x - z} = \frac{y}{z}$, **prove that :** $y \propto z$

2016 Exam (1) Question (5) (a)

73 **Calculate the standard deviation for the next data :**

16 , 32 , 5 , 20 , 27 *(El-Ismailia 2012)* « 9.3 »

74 **The following frequency distribution shows the number of children of some families in a new city :** *(Souhag 2013)*

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children. « 2 , 1 »

75 **Calculate the mean and the standard deviation for the following frequency distribution :** *(El-Beheira 2012 , Qena 2011)*

Set	0 –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	25

« 11.6 , 5.7 »

Essay Problems Answers

Problem number [1]

$\because (2x , 4) = (8 , y + 1)$

$\therefore 2x = 8 \qquad \therefore x = 4$

$, y + 1 = 4 \qquad \therefore y = 3$

$\therefore \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Problem number [2]

$\because (a - 7 , 26) = (-2 , b^3 - 1)$

$\therefore a - 7 = -2 \qquad \therefore a = -2 + 7 \qquad \therefore a = 5$

$, b^3 - 1 = 26 \qquad \therefore b^3 = 26 + 1$

$\therefore b^3 = 27 \qquad \therefore b = \sqrt[3]{27} = 3$

Problem number [3]

$\because (x - 1 , 11) = (8 , y + 3)$

$\therefore x - 1 = 8 \qquad \therefore x = 9$

$, y + 3 = 11 \qquad \therefore y = 8$

$\therefore \sqrt{x + 2y} = \sqrt{9 + 16} = \sqrt{25} = 5$

Problem number [4]

$\because (x^2 , 27) = (1 , y^3)$

$\therefore x^2 = 1 \qquad \therefore x = -1$ or $x = 1$ (refused)

$, y^3 = 27 \qquad \therefore y = 3$

$\therefore \sqrt{y - x} = \sqrt{3 + 1} = \sqrt{4} = 2$

Problem number [5]

$X \times Y = \{(1 , 2) , (1 , 3) , (1 , 4) , (2 , 2) , (2 , 3) , (2 , 4)\}$

Problem number [6]

$X^2 = \{(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)\}$

Problem number [7]

(1) $X \times Y = \{(2 , 1) , (2 , 3) , (2 , 7) , (5 , 1) , (5 , 3) , (5 , 7)\}$

(2) $n(Y^2) = 9$

Problem number [8]

(1) $X = \{1\} , Y = \{1 , 3 , 5\}$

(2) $Y \times X = \{(1 , 1) , (3 , 1) , (5 , 1)\}$

Problem number [9]

(1) $X = \{2\} , Y = \{3 , 2 , 4\}$

(2) $X \times (X \cap Y) = \{2\} \times \{2\} = \{(2 , 2)\}$

Problem number [10]

(1) $X \cup Y = \{1 , 2\} \cup \{2 , 3\} = \{1 , 2 , 3\}$

(2) $Y^2 = \{2 , 3\} \times \{2 , 3\}$

$= \{(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)\}$

Problem number [11]

(1) $Y \times Z = \{(4 , 15) , (1 , 15)\}$ (2) $n(X^2) = 4$

(3) $(X \cap Z) \times Y = \{15\} \times \{4 , 1\}$

$= \{(15 , 4) , (15 , 1)\}$

Problem number [12]

$(X \cap Y) \times (Y \cup Z) = \{2\} \times \{2 , 4 , 5\}$

$= \{(2 , 2) , (2 , 4) , (2 , 5)\}$

Problem number [13]

(1) $X \times (Y \cap Z) = \{3 , 4\} \times \{5\}$

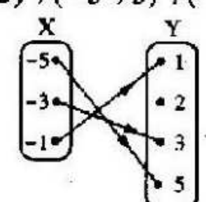
$= \{(3 , 5) , (4 , 5)\}$

(2) $(X - Y) \times Z = \{3\} \times \{6 , 5\}$

$= \{(3 , 6) , (3 , 5)\}$

Problem number [14]

(1) $R = \{(-5 , 5) , (-3 , 3) , (-1 , 1)\}$

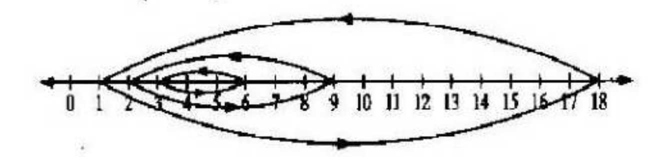


(2) R is a function because every element of X has only one image in Y

The range = $\{1 , 3 , 5\}$

Problem number [15]

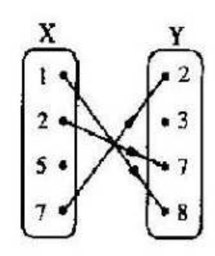
R = {(1 , 18) , (2 , 9) , (3 , 6) , (6 , 3) , (9 , 2) , (18 , 1)}



Problem number [16]

(1) R = {(1 , 8) , (2 , 7) , (7 , 2)}

(2)



(3) R is not a function because the element 5 ∈ X has no image in Y

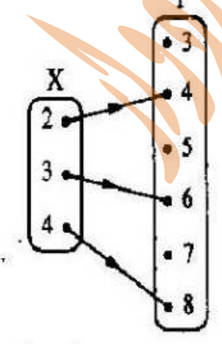
Problem number [17]

The images of the elements of X by the function f are 7 , 6 , 5

Problem number [18]

(1) R = {(2 , 4) , (3 , 6) , (4 , 8)}

(2)



(3) R is a function because every element of X has only one image in Y

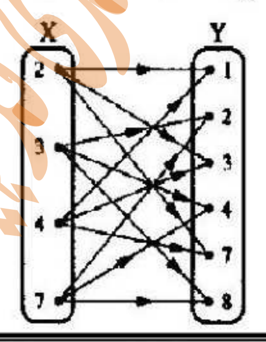
Problem number [19]

R = {(2 , 3) , (2 , 9) , (6 , 3) , (6 , 9) , (8 , 3) , (8 , 9)}

R is not a function because each of 2 ∈ X , 6 ∈ X , 8 ∈ X has more than one image in Y

Problem number [20]

R = {(2 , 1) , (2 , 3) , (2 , 7) , (3 , 2) , (3 , 4) , (3 , 8) , (4 , 1) , (4 , 3) , (4 , 7) , (7 , 2) , (7 , 4) , (7 , 8)}

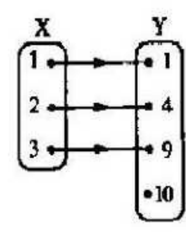


Problem number [21]

R = {(2 , 2) , (2 , 4) , (2 , 7) , (2 , 8) , (3 , 1) , (3 , 3) , (3 , 7) , (4 , 2) , (4 , 4) , (4 , 8) , (7 , 1) , (7 , 2) , (7 , 3) , (7 , 7) , (7 , 8)}

Problem number [22]

(1) R = {(1 , 1) , (2 , 4) , (3 , 9)}



(2) R is a function because every element of X has only one image in Y

The range = {1 , 4 , 9}

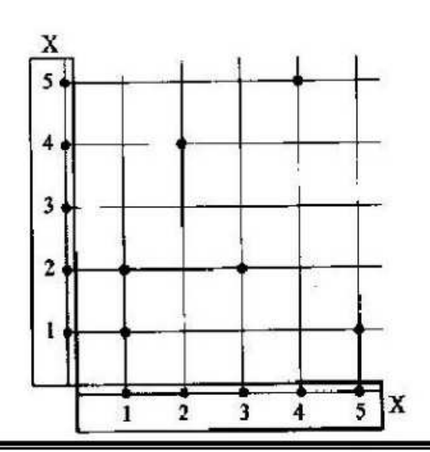
Problem number [23]

(1) The domain = {1 , 2 , 3 , 4 , 5} , The range = {3 , 5 , 7 , 9 , 11}

(2) $f(x) = 2x + 1$

Problem number [24]

$R = \{(1, 1), (1, 2), (2, 4), (3, 2), (4, 5), (5, 1)\}$



Problem number [25]

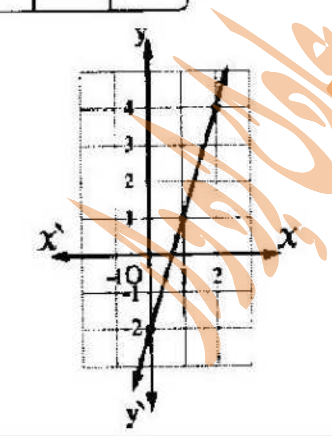
$R = \{(-3, 1), (-3, 8), (1, 2), (4, 4)\}$

R is not a function because the element $-3 \in X$ has two images in Y

Problem number [26]

$f(x) = 3x - 2$

x	0	1	2
f(x)	−2	1	4



Problem number [27]

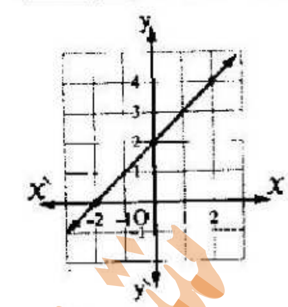
$\because$ (a , 5) lies on the straight line

$\therefore 5 = 3a - 4$ $\therefore 3a = 9$ $\therefore a = 3$

Problem number [28]

$f(x) = x + 2$

x	−2	0	2
f(x)	0	2	4



From the graph :

The area of the triangle $= \frac{1}{2} \times 2 \times 2 = 2$ square units

Problem number [29]

$\because$ The straight line cuts the y-axis at (b − 2 , 5)

$\therefore b - 2 = 0$ $\therefore b = 2$

$\because$ The straight line passes through the point (0 , 5)

$\therefore 5 = 2 \times 0 - a$ $\therefore a = -5$

$\therefore 3a + 2b = 3 \times (-5) + 2 \times 2 = -15 + 4 = -11$

Problem number [30]

$\because$ The straight line cuts the y-axis at (b , 3)

$\therefore b = 0$

, $\because$ The straight line passes through the point (0 , 3)

$\therefore 3 = 6 \times 0 - a$ $\therefore a = -3$

$\therefore 2a + 7b = 2 \times (-3) + 7 \times 0 = -6$

Problem number [31]

(1) $f\left(\frac{2}{3}\right) = 3 \times \frac{2}{3} + 4 = 2 + 4 = 6$

(2) $\because$ (a , −5) lies on the straight line

$\therefore -5 = 3a + 4$ $\therefore -9 = 3a$

$\therefore a = -3$

Problem number [32]

(1) $\because$ AO = 3 units $\quad \therefore$ A (3 , 0)

, $\because$ r is a constant function and passes through the point B

$\therefore$ B (0 , 6)

, $f(x)$ is a linear function and passes through A (3 , 0) , B (0 , 6)

$\therefore$ The rule of the function f is $f(x) = a\,x + b$

where $a = \frac{6-0}{0-3} = -2$

, b = 6 $\quad \therefore f(x) = -2x + 6$

(2) $f(6) + r(1) = 6 - 2 \times 6 + 6 = 0$

Problem number [33]

$f(-2) = (-2)^2 - (-2) + 3 = 4 + 2 + 3 = 9$

, $f(0) = (0)^2 - 0 + 3 = 3$

, $f(\sqrt{3}) = (\sqrt{3})^2 - \sqrt{3} + 3 = 3 - \sqrt{3} + 3 = 6 - \sqrt{3}$

Problem number [34]

(1) $f(\sqrt{3}) + 3\,g(\sqrt{3})$

$= (\sqrt{3})^2 - 3\sqrt{3} + 3(\sqrt{3} - 3)$

$= 3 - 3\sqrt{3} + 3\sqrt{3} - 9 = -6$

(2) $\because f(3) = 3^2 - 3 \times 3 = 9 - 9 = 0 \quad (1)$

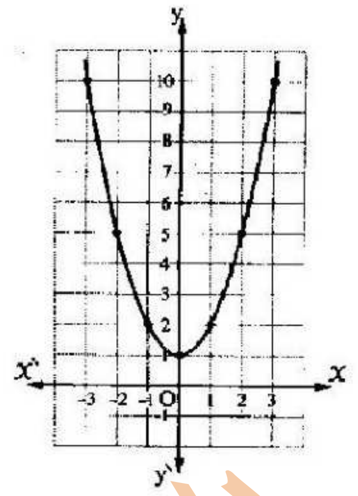
, $g(3) = 3 - 3 = 0 \quad (2)$

From (1) and (2) : $\therefore f(3) = g(3) = 0$

Problem number [35]

$f(x) = x^2 + 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	10	5	2	1	2	5	10



From the graph :

(1) The coordinates of the vertex of the curve is : (0 , 1)

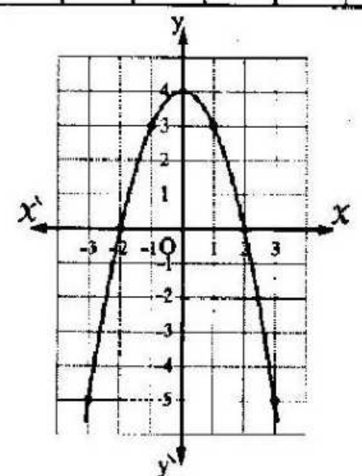
(2) The equation of the axis of symmetry is : $x = 0$

(3) The minimum value = 1

Problem number [36]

$f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-5	0	3	4	3	0	-5



From the graph :

(1) The vertex of the curve is : (0 , 4)

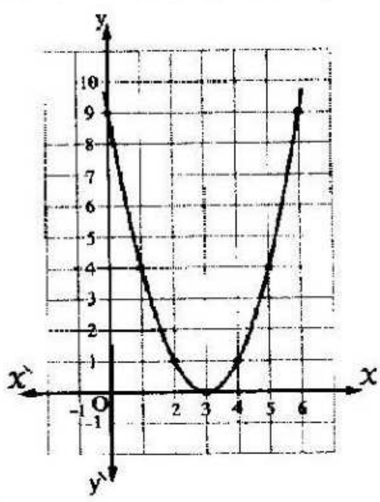
(2) The equation of the axis of symmetry is : $x = 0$

(3) The maximum value = 4

Problem number [37]

$f(x) = x^2 - 6x + 9$

x	0	1	2	3	4	5	6
$f(x)$	9	4	1	0	1	4	9



From the graph :

(1) The vertex of the curve is : (3 , 0)

(2) The minimum value = 0

(3) The equation of the axis of symmetry is : $x = 3$

Problem number [38]

$\because$ The curve cuts the x-axis at (– 2 , b) $\therefore b = 0$

$\because$ The curve passes through the point (–2 , 0)

$\therefore 0 = m - (-2)^2$

$\therefore 0 = m - 4 \qquad \therefore m = 4$

$\therefore m^b + 2m = 4^0 + 2 \times 4 = 1 + 8 = 9$

Problem number [39]

(1) $\because$ A O = 4 units $\qquad \therefore$ A (0 , 4)

$\because$ A (0 , 4) belongs to the curve of the function f

$\therefore$ A satisfies the equation of the curve

$\therefore 4 = m - (0)^2 \quad \therefore m = 4$

(2) $\because$ The curve of the function intersects x-axis at the two points B and C

$\therefore 0 = 4 - x^2 \quad \therefore x^2 = 4$

$\therefore x = 2$ or $x = -2$

$\therefore$ B = (2 , 0) , C = (– 2 , 0)

Problem number [40]

$\because \frac{x}{y} = \frac{2}{5} \qquad \therefore x = 2m , y = 5m$

$\therefore \frac{2x + y}{x + 4y} = \frac{4m + 5m}{2m + 20m} = \frac{9m}{22m} = \frac{9}{22}$

Problem number [41]

$\because \frac{x}{y} = \frac{2}{3} \qquad \therefore x = 2m , y = 3m$

$\therefore \frac{x + y}{5x - y} = \frac{2m + 3m}{10m - 3m} = \frac{5m}{7m} = \frac{5}{7}$

Problem number [42]

$\because \frac{a}{b} = \frac{c}{d} = m$

$\therefore a = bm , c = dm$

$\therefore \frac{a}{b - a} = \frac{bm}{b - bm} = \frac{bm}{b(1 - m)} = \frac{m}{1 - m} \qquad (1)$

$, \frac{c}{d - c} = \frac{dm}{d - dm} = \frac{dm}{d(1 - m)} = \frac{m}{1 - m} \qquad (2)$

From (1) and (2) : $\therefore \frac{a}{b - a} = \frac{c}{d - c}$

Problem number [43]

$\because 5a = 2b \qquad \therefore \frac{a}{b} = \frac{2}{5}$

$\therefore a = 2m , b = 5m$

$\therefore \frac{3a + 2b}{4a + 5b} = \frac{6m + 10m}{8m + 25m} = \frac{16m}{33m} = \frac{16}{33}$

Problem number [44]

$\because \frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m$

$\therefore a = 4m , b = 5m , c = 3m$

$\therefore$ L.H.S. $= \frac{a - b + c}{a + b - c} = \frac{4m - 5m + 3m}{4m + 5m - 3m} = \frac{2m}{6m}$

$= \frac{1}{3} =$ R.H.S.

Problem number [45]

$\because \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore x = 3m , y = 4m , z = 5m$

$\therefore 3x^2 + 3y^2 + z^2$

$= 3 \times 9m^2 + 3 \times 16m^2 + 25m^2$

$= 27m^2 + 48m^2 + 25m^2 = 100m^2 \qquad (1)$

$, (2x + y)^2 = (6m + 4m)^2 = (10m)^2 = 100m^2 \qquad (2)$

From (1) and (2) : $\therefore 3x^2 + 3y^2 + z^2 = (2x + y)^2$

Problem number [46]

$\because a : b : c = 5 : 7 : 3$

$\therefore a = 5\,m$, $b = 7\,m$, $c = 3\,m$

, $\because a + b = 24$ $\therefore 5\,m + 7\,m = 24$

$\therefore 12\,m = 24$ $\therefore m = 2$

$\therefore a = 5 \times 2 = 10$, $b = 7 \times 2 = 14$, $c = 3 \times 2 = 6$

Problem number [47]

$\because \frac{x}{2} = \frac{y}{3} = \frac{y - x}{1}$ $\therefore 5k = 1$ $\therefore k = \frac{1}{5}$

Problem number [48]

$\because \frac{a}{b - a} = \frac{c}{d - c}$ $\therefore a(d - c) = c(b - a)$

$\therefore ad - ac = bc - ac$

$\therefore ad = bc$ $\therefore \frac{a}{b} = \frac{c}{d}$

$\therefore$ a , b , c and d are proportional

Problem number [49]

$\because \frac{x + y}{5} = \frac{y + z}{3} = \frac{z + x}{6}$

, subtracting the antecedent and consequent of the 2nd ratio from the antecedent and consequent of the 1st ratio

$\therefore \frac{x + y - y - z}{5 - 3} = \frac{x - z}{2}$ = one of the given ratios (1)

, adding the antecedents and consequents of the three ratios

$\therefore \frac{x + y + y + z + z + x}{5 + 3 + 6} = \frac{2x + 2y + 2z}{14} = \frac{x + y + z}{7}$

= one of the given ratios (2)

From (1) and (2) : $\therefore \frac{x - z}{2} = \frac{x + y + z}{7}$

Problem number [50]

Let the two numbers be $2x$ and $3x$

$\therefore \frac{2x + 7}{3x - 12} = \frac{5}{3}$ $\therefore 6x + 21 = 15x - 60$

$\therefore 21 + 60 = 15x - 6x$

$\therefore 81 = 9x$ $\therefore x = 9$

$\therefore$ The two numbers are : 18 and 27

Problem number [51]

Let the number be x

$\therefore 7 + x$, $9 + x$, $12 + x$

, $15 + x$ are proportional

$\therefore \frac{7 + x}{9 + x} = \frac{12 + x}{15 + x}$

$\therefore (7 + x)(15 + x) = (9 + x)(12 + x)$

$\therefore 105 + 22x + x^2 = 108 + 21x + x^2$

$\therefore 22x - 21x = 108 - 105$ $\therefore x = 3$

$\therefore$ The number is : 3

Problem number [52]

Let the number be x

$\therefore 16 - x$, $21 - x$, $14 - x$, $18 - x$ are proportional

$\therefore \frac{16 - x}{21 - x} = \frac{14 - x}{18 - x}$

$\therefore (16 - x)(18 - x) = (21 - x)(14 - x)$

$\therefore 288 - 34x + x^2 = 294 - 35x + x^2$

$\therefore -34x + 35x = 294 - 288$

$\therefore x = 6$

$\therefore$ The number is : 6

Problem number [53]

$\because \frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$\therefore \frac{b}{b + c} = \frac{cm}{cm + c} = \frac{cm}{c(m + 1)} = \frac{m}{m + 1}$ (1)

, $\frac{a}{a + b} = \frac{cm^2}{cm^2 + cm} = \frac{cm^2}{cm(m + 1)} = \frac{m}{m + 1}$ (2)

From (1) and (2) : $\therefore \frac{b}{b + c} = \frac{a}{a + b}$

Problem number [54]

$\because$ b is the middle proportion between a and c

$\therefore b^2 = ac$

$\therefore \text{L.H.S.} = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c} = \text{R.H.S}$

Problem number [55]

$\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{ab - cd}{b^2 - c^2} = \frac{dm^3 \times dm^2 - dm \times d}{d^2 m^4 - d^2 m^2} = \frac{d^2 m (m^4 - 1)}{d^2 m^2 (m^2 - 1)}$

$= \frac{(m^4 - 1)}{m (m^2 - 1)} = \frac{(m^2 - 1)(m^2 + 1)}{m (m^2 - 1)}$

$= \frac{m^2 + 1}{m}$ (1)

, $\frac{a + c}{b} = \frac{dm^3 + dm}{dm^2} = \frac{dm (m^2 + 1)}{dm^2} = \frac{m^2 + 1}{m}$ (2)

From (1) and (2) : $\therefore \frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

Problem number [56]

$\because \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$

$\therefore a^2c^2 + b^2c^2 = b^4 + b^2c^2$

$\therefore b^4 = a^2c^2$ $\therefore b^2 = a c$

$\therefore$ b is the middle proportional between a and c

Problem number [57]

Let the number be x :

$\therefore \frac{1 + x}{7 + x} = \frac{7 + x}{25 + x}$

$\therefore (7 + x)^2 = (1 + x)(25 + x)$

$\therefore x^2 + 14x + 49 = x^2 + 26x + 25$

$\therefore 12x = 24$ $\therefore x = 2$ $\therefore$ The number is 2

Problem number [58]

(1) $\because y \propto x$ $\therefore y = m x$ $\therefore 21 = 7 m$

$\therefore m = 3$ $\therefore y = 3x$

(2) at $x = 4$ $\therefore y = 3 \times 4$ $\therefore y = 12$

Problem number [59]

$\because y \propto x$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$ $\therefore \frac{\frac{5}{3}}{\frac{3}{4}} = \frac{\frac{1}{6}}{x_2}$

$\therefore x_2 = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{5}{3}} = \frac{3}{40}$

Problem number [60]

(1) $\because y \propto \frac{1}{x}$ $\therefore xy = m$ $\therefore 3 \times 5 = m$

$\therefore m = 15$ $\therefore xy = 15$

(2) at $x = 7.5$ $\therefore 7.5 y = 15$ $\therefore y = 2$

Problem number [61]

$\because y \propto \frac{1}{x}$ $\therefore xy = m$ $\therefore 3 \times 10 = m$

$\therefore m = 30$ $\therefore xy = 30$

at $x = 5$ $\therefore 5y = 30$ $\therefore y = 6$

Problem number [62]

$\because w \propto r$ $\therefore \frac{w_1}{w_2} = \frac{r_1}{r_2}$

$\therefore \frac{182}{312} = \frac{35}{r_2}$ $\therefore r_2 = \frac{312 \times 35}{182} = 60$ kg.

Problem number [63]

$\because v \propto \frac{1}{r^2}$ $\therefore \frac{v_1}{v_2} = \frac{r_2^2}{r_1^2}$

$\therefore \frac{5}{v_2} = \frac{(15\frac{3}{4})^2}{(3^2)}$ $\therefore \frac{5}{v_2} = \frac{248.0625}{9}$

$\therefore v_2 = \frac{5 \times 9}{248.0625} = \frac{80}{441}$ cm./sec.

Problem number [64]

(1) $\because y = 5 + a$, $a \propto x$ $\therefore a = m x$

$\therefore 6 = 2m$ $\therefore m = 3$

$\therefore a = 3x$ $\therefore y = 5 + 3x$

(2) at $y = 8$

$\therefore 8 = 5 + 3x$ $\therefore 3x = 8 - 5$

$\therefore 3x = 3$ $\therefore x = 1$

Problem number [65]

(1) $\because y = a + 7$, $a \propto \frac{1}{x^2}$ $\quad \therefore a = \frac{m}{x^2}$

$\therefore 3 = \frac{m}{4}$ $\quad \therefore m = 12$ $\quad \therefore a = \frac{12}{x^2}$

$\therefore y = \frac{12}{x^2} + 7$

(2) at $x = \sqrt{3}$

$\therefore y = \frac{12}{(\sqrt{3})^2} + 7$ $\quad \therefore y = \frac{12}{3} + 7$

$\therefore y = 4 + 7$ $\quad \therefore y = 11$

Problem number [66]

(1) The variation is inverse

(2) $\because y \propto \frac{1}{x}$ $\quad \therefore y\,x = m$ $\quad \therefore m = 12$

(3) at $x = 3$ $\quad \therefore 3y = 12$ $\quad \therefore y = 4$

Problem number [67]

$\because x^2 y^2 - 8xy + 16 = 0$ $\quad \therefore (xy - 4)^2 = 0$

$\therefore xy - 4 = 0$ $\quad \therefore xy = 4$

$\therefore x = \frac{4}{y}$ $\quad \therefore x \propto \frac{1}{y}$

Problem number [68]

$\because x^2 + 4y^2 = 4xy$ $\quad \therefore x^2 - 4xy + 4y^2 = 0$

$\therefore (x - 2y)^2 = 0$ $\quad \therefore x - 2y = 0$

$\therefore x = 2y$ $\quad \therefore x \propto y$

Problem number [69]

$\because a^2 b^4 - 10ab^2 + 25 = 0$ $\quad \therefore (ab^2 - 5)^2 = 0$

$\therefore ab^2 - 5 = 0$ $\quad \therefore ab^2 = 5$

$\therefore a = \frac{5}{b^2}$ $\quad \therefore a \propto \frac{1}{b^2}$

Problem number [70]

$\because a^2 b^2 - ab + \frac{1}{4} = 0$

$\therefore \left(ab - \frac{1}{2}\right)^2 = 0$ $\quad \therefore ab - \frac{1}{2} = 0$

$\therefore ab = \frac{1}{2}$ $\quad \therefore a \propto \frac{1}{b}$

Problem number [71]

$\because 4x^2 - 12xy + 9y^2 = 0$

$\therefore (2x - 3y)^2 = 0$ $\quad \therefore 2x - 3y = 0$

$\therefore 2x = 3y$ $\quad \therefore x = \frac{3}{2} y$

$\therefore x \propto y$

Problem number [72]

$\because \frac{21x - y}{7x - z} = \frac{y}{z}$

$\therefore 7xy - yz = 21xz - yz$

$\therefore 7xy = 21xz$ $\quad \therefore y = 3z$

$\therefore y \propto z$

Problem number [73]

The mean $(\bar{x}) = \frac{16 + 32 + 5 + 20 + 27}{5} = 20$

x	$x - \bar{x}$	$(x - \bar{x})^2$
16	$16 - 20 = -4$	16
32	$32 - 20 = 12$	144
5	$5 - 20 = -15$	225
20	$20 - 20 = 0$	0
27	$27 - 20 = 7$	49
Total		434

The standard deviation $(\sigma) = \sqrt{\frac{434}{5}} \simeq 9.3$

Problem number [74]

Number of children (x)	Number of families (k)	$x \times k$
0	8	0
1	16	16
2	50	100
3	20	60
4	6	24
Total	100	200

The mean of $\left(\overline{x}\right) = \frac{200}{100} = 2$ children

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	8	$0 - 2 = -2$	4	32
1	16	$1 - 2 = -1$	1	16
2	50	$2 - 2 = 0$	0	0
3	20	$3 - 2 = 1$	1	20
4	6	$4 - 2 = 2$	4	24
Total	100			92

The standard deviation $(\sigma) = \sqrt{\frac{92}{100}} \simeq 1$ child

Problem number [75]

Sets	centres of sets (x)	Frequency (k)	$x \times k$
0 –	2	3	6
4 –	6	4	24
8 –	10	7	70
12 –	14	2	28
16 – 20	18	9	162
Total		25	290

The mean of $\left(\overline{x}\right) = \frac{290}{25} = 11.6$

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
2	3	$2 - 11.6 = -9.6$	92.16	276.48
6	4	$6 - 11.6 = -5.6$	31.36	125.44
10	7	$10 - 11.6 = -1.6$	2.56	17.92
14	2	$14 - 11.6 = 2.4$	5.76	11.52
18	9	$18 - 11.6 = 6.4$	40.96	368.64
Total	25			800

The standard deviation $(\sigma) = \sqrt{\frac{800}{25}} \simeq 5.7$

Part One Questions

Complete each of the following:-

1) The point (5 , -3) lies in …………… quadrant.

2) If (x + 5 , 8) = (1 , 6y + x) then x = ………… , y = ……….

3) If n (X) = 5, n (X x Y) = 15 then n (Y) = …………

4) The point (4 , 0) lies on ………. axis.

5) If (5 , x-7) = (y + 1, -5) then x + y = …………

6) If X x Y = {(1 , 5) , (1 , 7) , (2 , 5) , (2 , 7) , (3 , 5) , (3 , 7)}

then X = ………… Y = …………

7) If f (x) = 5 x -7 then f (3) = …………

8) If f (x) = 6 x then f (2) + f (-2) = …………

9) Function f : R → R such that f (x) = 3 x represented by a straight line passes through the point (-4 , …………)

10) The linear function f(x) = x + 7 is represented by a straight line cuts X – axis at the point ……………

11) The linear function f (x) = x + 7 is represented by a straight line cuts X- axis at the point …………

12) The linear function f (x) = 2 x- 1 is represented by a straight line cuts y – axis at the point …………

13) If the point (a , 3) lies on the straight line which represents the function f : R → R where f(x) = x-5 then a = …………

14) If f (x) = x- 6 and $\frac{1}{3}$ f (a) = -2 then a = …………

15) If X = {1 , 3 , 5} , f : X → R and f (x) = 2x + 1 then the range of f = ……….

16) The linear function f(x) = 2-3x is represented by a straight line cuts Y – axis at the point ………….

17) If f is function where f : X → Y then X is called ……… and Y is called ………….

18) If f is function from set X to set Y then the range of function f C : ……………

19) f is a function where f(x) = 3x – 1 is represented graphically by a line passes through point (a , 2) then a = …………

20) (2 , -4) ∈ f (x) where f (x) = kx + 8 then k =………..

Choose the correct answer from those given:-

1) If n (x^2) = 9 , then n (X) = …………

a) 3 b) 6 c) 18 d) 81

2) The point (-3 , 4) lies in ………… Quadrant.

a) first b) second c) third d) fourth

3) X = {5 , 6 , 7} then n (X^2) = ……………

a) 3 b) 6 c) 9 d) 12

4) of X x Y {(1 , 3) , (1 , 4)} then n (X) = ...

a) 3 b) 1 c) 4 d) 2

5) If X = {5} Y = {3} then n (X x Y)

a) 15 b) 8 c) 2 d) 1

6) If X = {3 , 5 , 7} and R is a relation on X then the relation which represents a function is ....

a) R = {(3 , 5) , (5 , 3) , (3 , 7)}
b) R = {(3 , 5) , (5 , 7)}
c) R = {(3 , 5) , (5 , 5) , (7 , 5)}
d) R = {(3 , 3) , (3 , 5) , (3 , 7)}

7) If the point (x, 7) lies on Y = axis then 5x + 1 = ...

a) zero b) 1 c) 5 d) 6

8) If R is a function from set X to set Y where

X = {2 , 5 , 8} , Y = { 3, 5} and R = {(2 , 3) , (5 , 3) , (x , 3)} then x =.........

a) 2 b) 3 c) 5 d) -8

9) If R is a function where R = {(4, 3) (5 , 6) , (9 , 3)} then the range of the function R is ....

a) {3 , 4 , 5 , 6 , 9}
b) {4 , 5 , 9}
c) {3 , 6 , 9}
d) {3 , 9}

10) If f (x) = 7x - $\frac{1}{2}$ then f ($\frac{1}{2}$) = ...

a) 7 b) $\frac{1}{2}$ c) $\frac{7}{2}$ d) 3

11) If f (x) = 4x + b f (3) = 15 then b = ...

a)156 b) 3 c) 4 d) -3

12) If (m , 13) satisfies the function f where f (x) = 3x + 4 then m = ...

a) 6 b) -6 c) 3 d) -3

13) If (2 , b) satisfies the function f where f(x) = 3x – 6 , then b = ...

a) Zero b) 7 c) 9 d) 2

14) If f (x) = x^2 + 7 then f (3) = ...

a) 10 b) 7 c) 9 d) 16

15) If f (x) = x^3 then f (2) + f (-2) = ...

a) 16 b) Zero c) -7 d) 4

16) if (2 , -6) satisfies the function f where f(x) = k x + 8 then k = ...

a) -16 b) 7 c) -7 d) 2

17) The function f, where f (x) = 5 x is represented graphically by a straight line passes through the point.....

a) (5 , 5) b) (0 , 0) c) (0 , 5) d) (5, 0)

18) If f (x) = 5x + 4 is represented graphically by a straight line passes through the point (3 , b) then b =......

a) 5 b) 4 c) 3 d) 19

19) If the function f is a function from set X to set Y then the domain of the function is ....

a) X b) Y c) X x Y d) Y x X

Answer the following questions:-

1) If X = {0 , 1 , 2 , 3 , 4 , 5 , 6} and R is a relation on X where aRb means "a is twice b" for all a , b ∈ X., a ≠ b .

a) Write R and represent it by an arrow diagram.

b) Is (0 , 0) ∈ R ?

c) Is 2R4 ?

d) Find x If 6Rx

2) If X = {2 , 4 , 8 }, Y= {4 , 6 , 12 , 24}, and R is a relation from X to Y such that aRb means "b > 2a" for all a ∈ X , b ∈ Y, write R and represent it by an arrow diagram and by a Cartesian diagram.

3) If X = {13 , 14 , 43 , 84}, and R is a relation on X such that aRb means "two numbers a and b have the same unit digit" for all a , b, ∈ X. write R and represent it on a lattice.

4) If X = {2 , 3 , 4 , 7}, Y = {1 , 2, 3 , 4 , 7 , 8} and R is a relation from X to Y where aRb means "a -b is a prime number" for all a ∈ X, b ∈ Y an arrow diagram.

5) If X = {0, 1, 2 , 3} , Y = {-3 , -2 , -1 , 0}and R is a relation from X to Y where aRb means "a is additive –inverse of b" for all a ∈ X , b ∈ Y, write R and represent it by an arrow diagram and graphically. It R ia a function? Why?

6) If X = {2 , 5 , 8} , Y = {10 , 16 , 24 , 30} and R is a relation from X to Y for all a ∈ X, b ∈ Y where "a is factor of b" write R and represent it by an arrow diagram. Is R a function? Why?

7) If X = {1 , 3 , 4 , 5}, Y = {1 , 2 , 3 , 4, 5, 6} and R is a relation from X to Y where aRb means "a + b = 7" for all a ∈ X, b ∈ Y, write and represent it by an arrow diagram and by a Cartesian diagram, show that R is a function? Write its domain and its range.

8) If X = {2 , 3 , 4} , Y = {3 , 4 , 5 , 6 , 7 , 8} and f: X → Y where f(x) = 9-x find the images of the elements of x and represent it by an arrow diagram.

9) If X = {1 , 3 , 5} and R is a relation on X where R = {(a , 3) , (b , 1) , (1 , 5)} then find the numerical value of the expression a + b.

10) Graph the function f where $f(x) = 4 - x^2$ in the interval [-3 , 3], from the graph determine:

First : The coordinates of the maximum value of function.

Second : The equation of the axis of symmetry.

11) Graph the function f , where f(x) = x (6-x) + 4 in the interval [-1, 7].

12) Represent the following linear functions graphically. f(x) = 3x + 1

13) If the straight line which represents the function f: R → R, where f(x) = 6x – a cuts Y – axis at the point (b, 3) then find the value of a and b

14) If X = {3, 4 , 5 , 10 , 13} , Y = {4 , 5 , 7 , 8 , 9 , 19 , 25} and R is a relation from X to Y such aRb means "b = 2a – 1 " for all a ∈ X and b ∈ Y.

a) Write R

b) Represent R by a Cartesian diagram.

c) Find the value of x if (x , 9) ∈ R.

15) If X = {3, 5 , 7 , 9}, Y = {a : a ∈ N , 10 ≤ a < 50} and R is relation from X to Y, where R = {(3 , 15) , (5 , 25) , (7 , 35) (9 , 45)} . write the rule of R.

16) If X = {1 , 2 , 3}, Y = {1 , 3 , 6 , 9 , 13} and R is a relation from X to Y where means " a = $\frac{1}{3}$ b" for all a ∈ X , b ∈ Y. write R and show that it is a function, write its range.

17) If function f = {(1 , 3) , (2 , 5) , (3 , 7) , (4 , 9) , (5 , 11)} .

a) Write each of domain and range of f.

b) write the rule of the function f.

18) If f : R → R is represented by a straight line cuts Y-axis at (b, 3) where f(x) = 6x – a find value of 2a + 7b.

19) Find the value of x if $\frac{x+7}{x+11} = \frac{2}{3}$

20) Find the value of x if (2x + 3) : (9x – 3) = 6 : 5

21) Find the number which if added to the two terms of ratio 7 : 11 it will be 2 : 3

22) Two integers the ratio between them is 3 : 7 and if subtracted 5 from each terms the ratio between them because 1 : 3 find the two numbers.

Part One

Model Answers

Complete each of the following:-

1) 4^{th}
2) X = -4 , Y = 2
3) n (y) = 3
4) X – axis
5) X + Y = 2
6) X = {1 , 2 , 3}, Y = {5 , 7}
7) f(3) = 8
8) Zero.
9) b = 1
10) (-4 , -12)
11) (-7 , 0)
12) (0 , -1)
13) a = 8
14) a = zero.
15) Range = { 3, 7 11}.
16) (0 , 2)
17) Domain , Codomain.
18) Y
19) a = 1
20) K = -7

Choose the correct answer from those given:-

1) a
2) b
3) c
4) b
5) d
6) c
7) b
8) d
9) d
10) d
11) b
12) c
13) a
14) d
15) b
16) c
17) b
18) d
19) a

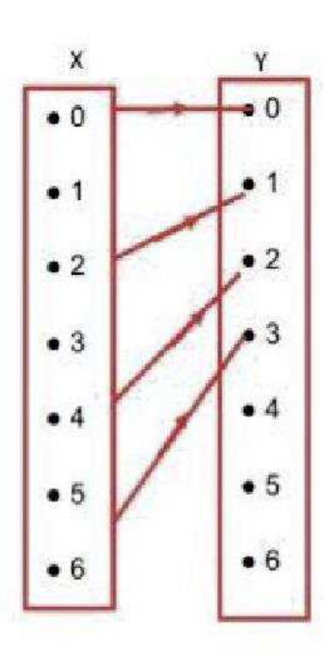
Answer the following questions:-

1) a) R = {(4 , 2) , (6 , 3)}

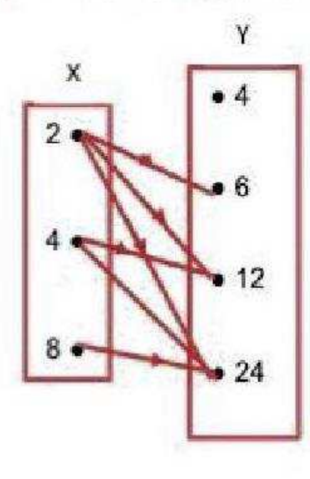
b) (0 , 0) ∉ R

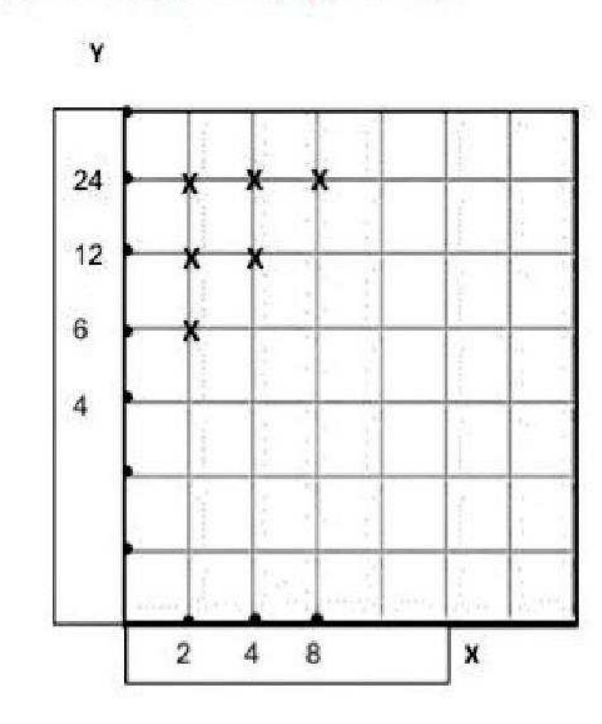
c) (2 , 4) ∉ R

d) (x = 3)

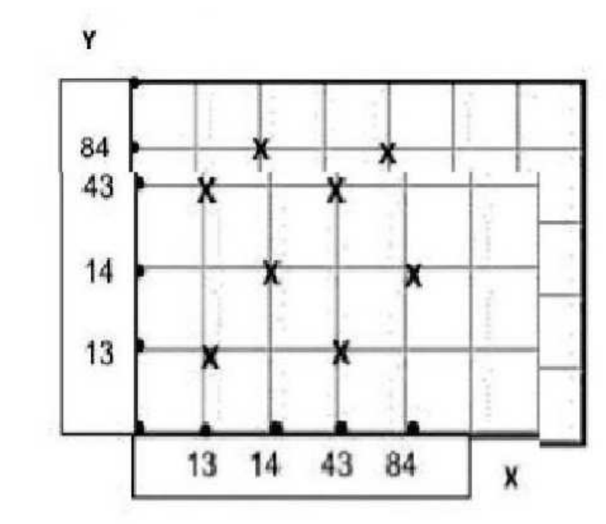


2) R = {(2 , 6) , (2 , 12) , (2 , 24) , (4 , 12) , (4 , 24) , (8 , 24)}

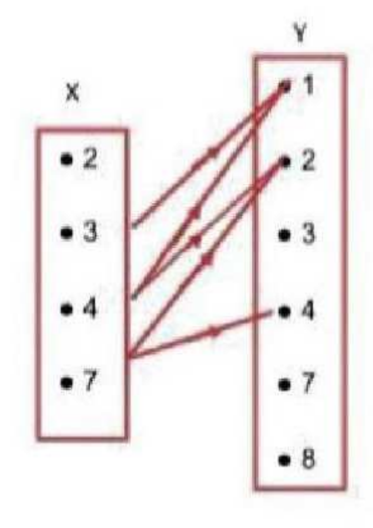




3) R = {(13 , 13) , (13 , 43) , (14 , 14) , (14 , 84), (43 , 13) , (43 , 43) , (84 , 14) , (84 , 84)}.



4) R = {(3 , 1) , (4 , 1) , (7 , 2) , (7 , 2) , (7 , 4) , (4 , 2) }



5) R = {(0, 0) , (1, -1) , (2 , -2) , (3 , -3) } , yes.

6) R = {(2 , 10) , (2 , 16) , (2 , 24) , (2 , 24) , (2 , 30) , (5 , 10) , (5 , 30) , (8 , 16) , (8 , 24)} No

7) R = {(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)}, R is a function from X → Y

range = { 6 , 4 , 3 , 2}

8) f(2) = 9 – 2 = 7

f(3) = 9 – 3 = 6

f(4) = 9 – 4 = 5

f = {(2, 7) , (3 , 6), (4 , 5)}.

9) R = {(5 , 3) , (3 , 1) , (1 , 5)}

a = 5   b = 3   ∴ a + b = 8

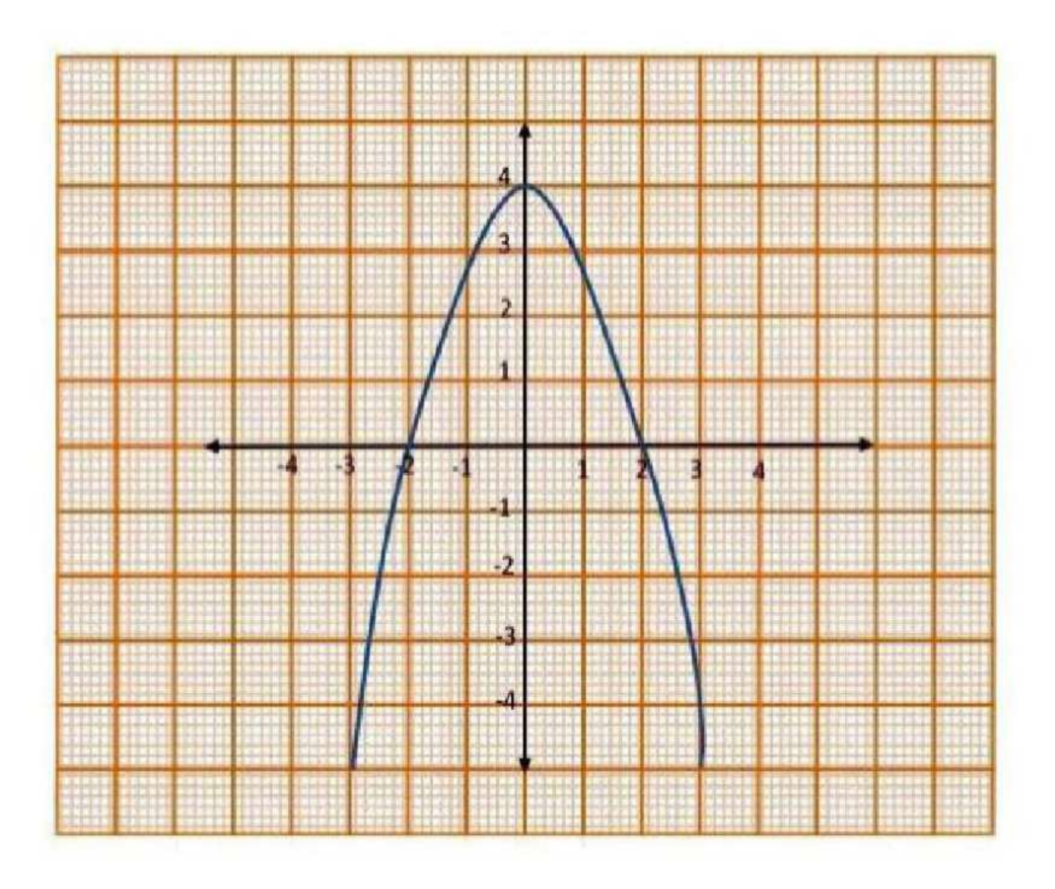
10) $f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
f(x)	-5	0	3	4	3	0	-5

Co-ordinate of max value (0 , 4)

Equation of the axis of symmetry x = 0

Maximum value = 4



11) $f(x) = x(6 - x) + 4 = 6x - x^2 + 4$ do it you self.

12) a) $f(x) = 3x + 1$

b) $f(0) = -3 + 1 = -2$

c) $f(-1) = 3 + 1 = 4$

represent it by you self.

x	-1	0	1
f(x)	-2	1	4

13) $f(x) = 6x - a$

∵ The 5 line cats y-axis at (b , 3), then b = 0 (0, 3) satis fies the equation of the s line.

$f(0) = 6(0) - a = 3$ $\rightarrow -a = 3$ $\rightarrow a = -3$

14) a) R = {(3 , 5) , (4 , 7) , (5 , 9) , (10 , 19) , (13 , 25)}

b) ∵ $b = 2a - 1$

∴ $9 = 2x - 1$ $\Rightarrow 2x = 10$ $\Rightarrow x = 5$

15) ∵ R = {(3 , 15) , (5 , 25) , (7 , 35), (9 , 45)}

3x5=15 5x5=25 7x5=35 9x5=45

The rule of R is b = 5a

16) R= {(1 , 3) , (2, 6) , (3 , 9) } , it's a function , range = {3 , 6 , 9}.

17) a) domain = {1 , 2 , 3 , 4 , 5} , Range = {3 , 5 , 7 , 9, 11}

(1 , 3) (2 , 5) (3 , 7) (4 , 9) (5 , 11)

1x2+1=3 2x2+1=5 3x2+1=7 4x2+1=9 5x2+1=11

The rule of the function is 2a + 1

18) (5 , 3) → b = 0 ⇒ (0 , 3) $f(x) = 6x - a \Rightarrow 3 = 6(0) - a$

a= -3 the value of 2a + 7b = 2(-3) + 7(0) = -6

19) $3(x+7) = 2(x+11)$

$3x + 21 = 2x + 22$

$X = 1$

20) $10X + 15 = 54X - 18$

$44X = 33$ $\rightarrow X = \frac{3}{4}$

21) $\frac{7+x}{11+x} = \frac{2}{3}$

$21 + 3X = 22 + 2X$

$X = 1$

22) let the first no 3X , let the second number 7X

$\frac{3x-5}{7x-5} = \frac{1}{3}$

$9X - 15 = 7X - 5$

$X = 5$

The first no. = 3× 5 = 15

The second no. = 7× 5 = 35

Part Two Questions

Complete the following:-

1) If 3 a = 4b then a : b = ... : ....

2) If 3 ,4, c and 8 are proportional then c = ...

3) The proportional mean of 3 a b^2 and 27 a^3b^2 is ....

4) If $\frac{a}{3} = \frac{y}{5}$, then $\frac{3a}{5y}$ = ...

5) If 9 , 2x , $\frac{1}{y^2}$ are proportional quantities then xy = ...

6) If 4 x^2 – 12 xy + 9y^2 = 0 and x ∈ R, y ≠ 0 then $\frac{x}{y}$ = $\frac{\dots\dots}{\dots\dots}$

7) If $\frac{a}{b} = \frac{2}{3}$ and $\frac{a}{c} = \frac{3}{5}$ then a : b : c = ... : .... : ....

8) If $\frac{a}{b} = \frac{7}{2}$ then $\frac{a-b}{a+b}$ = $\frac{\dots\dots}{\dots\dots}$

9) $\frac{x}{6} = \frac{y}{5} = \frac{z}{4} = \frac{\dots\dots}{11} = \frac{2y+2}{\dots\dots}$

10) If 1 , x , 9 , y are in continued proportion then x = ... Y = ...

2- Choose the correct answer from those given:

1- The third proportion of the two numbers 3 and 6 is ....

a) $\frac{1}{2}$ b) 2 c) 9 d) 12

2- If 2, 6 , x + 15 are proportional then x = ...

a) 1 b) 2 c) 3 d) 4

3- If a , b, 2 and 3 are proportional, then $\frac{a}{b}$ = ...

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

4- If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and b 0) then $\frac{a}{b}$ =.............

a) $\frac{2}{3}$ b) $\pm\frac{3}{2}$ c) $\pm\frac{2}{3}$ d) $\pm\frac{4}{3}$

5- The second proportion of the quantities 12ab^2 , ..., 21ab, 14b^2 .

a) 8ab^2 b) 8b^3 c) 24ab d) 24b^4

6- If $\frac{x}{y} = \frac{z}{\ell}$ which of the following is true ?

a) $\frac{x}{\ell} = \frac{y}{z}$ b) $\frac{x}{z} = \frac{\ell}{y}$ c) $\frac{x}{y} = \frac{\ell}{z}$ d) $\frac{x}{z} = \frac{y}{\ell}$

7- The number which added to each of the numbers 1,3,7,15 respectively to be in continued proportion is ....

a) 1 b) 2 c) 3 d) 4

8- If $\frac{a}{2} = \frac{b}{3}$ then $\frac{b-a}{b+a}$ equals ....

a) $\frac{1}{5}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$

9- If $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$, then z = ...

a) -2 b) -$\frac{1}{2}$ c) $\frac{1}{2}$ d) 2

10- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ = m (where m ∈ R*), then $\frac{ace}{bdf}$ equals ....

a) m b) 3m c) m^3 d) $3m^3$

3- If the following sets of numbers are proportional, then find the values of x.

a) 8, x, 4, 5 b) 11,3,x,6 c) 6, 24, 1, x

4- Find. x : y : z in each of the following.

a) $\frac{x}{y} = \frac{3}{5}$ and $\frac{y}{z} = \frac{4}{7}$ b) $\frac{x}{y} = \frac{4}{5}$ and $\frac{x}{z} = \frac{3}{7}$

5- If $\frac{a}{b} = \frac{2}{5}$, then find the value of each of the following ratios.

a) $\frac{a+b}{b}$ b) $\frac{a}{b-a}$ c) $\frac{b-a}{b+a}$ d) $\frac{7a-2b}{3a+2b}$

6- If $\frac{a}{2} = \frac{b}{3} = \frac{2a-b}{m}$, then find the value of m

7- If $\frac{a}{b-a} = \frac{c}{d-c}$, then prove that a , b, c and d are proportional.

8- If b is the middle proportional between a and c, then prove that.

a) $\frac{a^2}{b^2} = \frac{b^2}{c^2} = \frac{2a}{c}$ b) $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = b^2$

9- If $\frac{x}{3} = \frac{y}{4} = \frac{7}{5}$, then prove that :

a) $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$ b) $\sqrt{3x^2+3y^2+z^2} = 2x + y$

10- If a -1, a + 1 , b -2 , b +2 are in proportion, then find $\frac{a}{b}$, then prove that $\frac{a+b}{a+b-3} = \frac{3a}{5a-b-3}$

11- If $\frac{a}{b} = \frac{1}{3}$, $\frac{a}{c} = \frac{1}{9}$ and a + b + c = 26, then find each of a, b and c.

12- If x , y , z, ℓ are proportional quantities , then prove that:

a) $\frac{x+y}{z+\ell} = \frac{2x^2-3y^2}{2z^2-3\ell^2}$ b) $\sqrt[3]{\frac{5x^2-3y^2}{5y^3-3\ell^3}} = \frac{x+z}{y+\ell}$

13- If $\frac{x+y}{\ell+m} = \frac{y+z}{m+n} = \frac{z+x}{n+\ell}$, then prove that : $\frac{x}{\ell} = \frac{y-x}{m-\ell}$

14- If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, then prove that : $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

15- If $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$, then prove that : $\frac{x+y+z}{x-z} = 5$

16- Find the number that should be added to each of the numbers 7, 9, 12 , 15 to be proportional.

17- Two positive integer numbers, the ratio between them is 3 : 7 and if we subtract 5 from each of them the ratio becomes 1 : 3, find the two numbers.

18- Find the positive number that if we add its square to each term of the ratio 7 : 11 it becomes 4 : 5

Exercises:

1- Complete the following:

1- If y = 3x then y ∝ ....

2- If x y- 7 = 0 then y ∝ ....

3- If y ∝ x and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively then $\frac{x_1}{x_2} = \frac{\dots}{\dots}$

4- If y ∝ x and x =1 as y = 4 then the constant of variation is ....

5- If y ∝ x and y = 2 when x = 4 then y = ... x

6- If y varies inversely as x and y = 2 when x = $\frac{1}{2}$ then y = $\frac{\dots}{x}$

7- If $x^2y^2 - 4xy + 4 = 0$ then y ∝ ....

8- If $y^2 - 6xy + 9x^2 = 0$ then y ∝ ....

9- If $y \propto \frac{1}{x}$ and the variable x took the two values x_1 and x_2 and the variable y took the two values, y_1 and y_2 respectively then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$

10- If $y \propto x$ and y = 1 when x = 4 then y = ……. when x=8

Choose the correct answer from those given:

1- The relation which represents direct variation between the two variables x and y is ….

a) x y = 7 b) y=x+2 c) $\frac{x}{3} = \frac{4}{y}$ d) $\frac{x}{5} = \frac{y}{2}$

2- If y varies inversely as x and if $x = \sqrt{3}$ as $y = \frac{2}{\sqrt{3}}$, then the constant of variation equals.

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 2 d) 6

3- If $y - x = \frac{1}{x} - \frac{1}{y}$ where $x \neq Y \neq 0$ then

a) $y \propto x + 1$ b) $y \propto x$ c) $y \propto \frac{1}{x}$ d) $y \propto \frac{1}{x^2}$

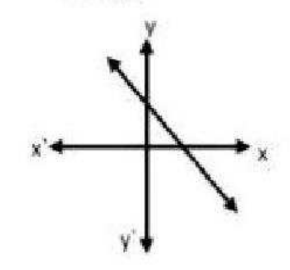
4- If the total cost (y) for a certain trip and if the some of this total cost is constant (a) and the other changes with the number of participated (x), which of the following relations is correct:

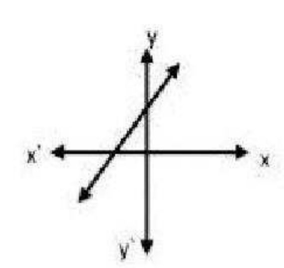
a) y = a x b) $y = \frac{a}{x}$

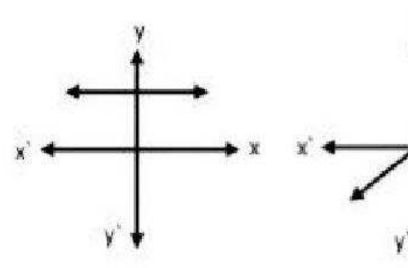
c) $y = a + \frac{m}{x}$, (m is a constant ≠ 0)

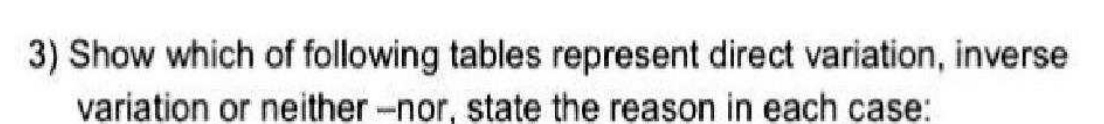
d) y = a + mx, (m is a constant ≠ 0)

5- The graph which represents the direct variation between x and y is ……..









3) Show which of following tables represent direct variation, inverse variation or neither –nor, state the reason in each case:

X	Y
3	20
5	12
4	15
6	10

X	Y
2	9
4	18
6	54
16	72

X	Y
5	9
10	18
15	27
25	45

X	Y
3	6
-2	-9
-18	1
9	-2

4) If y varies as x and y = 10 when x = 7 , find x when y = 20

5) If y varies inversely as x and y = 10 when x=3, find y when x=5

6) If $y \propto x$ and y = 20 when x =7 find the relation between x and y then find the value of y when x = 14

7) If $y \propto \frac{1}{x}$ and $x = 2\frac{4}{5}$ when $y = \frac{4}{7}$, then find the relation between x and y then find also the value of y when $x = 3\frac{1}{5}$

8) If y = 3 + a and $a \propto \frac{1}{x}$ if y = 5 when x =1, then find the relation between x and y and find y when x = 2.

9) Let y = a + 7 and $a \propto \frac{1}{x^2}$ if a = 18 when $x = \frac{2}{3}$ find the relation between y and x, then deduce the value of y when x = 6 .

10) If $\frac{21x-y}{7x-z} = \frac{y}{z}$, then prove that $y \propto z$

11) From the data of the following table answer the following questions.

X	2	4	6
Y	6	3	2

a) Identify the kind of variation whether it is direct or inverse.

b) Find the constant of variation.

c) Find the value of x when $y = 2\frac{2}{5}$

12) A car moves with constant velocity such that the covered distance varies directly as the time if the car covered 90 km within one and half an hour. Write down the relation between the covered distance and the time, then find the covered distance within $2\frac{1}{2}$ hours.

13) If the light tension (t) of a lamp varies inversely as the square of the distance (d) between the lamp and a pupil studies his lessons at a distance of 12 metres. If the tension of light is weak, then what is the distance which the lamp should be far from the pupil in order that the light tension becomes 4 times what it was before?

14) If the height of a right circular cylinder (h) of a constant volume varies inversely as the square of the radius length of its base (r) and h= 18cm when r=7cm. find the height h when r=10.5cm.

15) A car of mass 3 ton moves with uniform velocity under resistence varies as its velocity. If the resistence was 6kg. weight/ton of the mass of the car when the velocity was 50 km/H find the velocity of the car if the resistence becomes 27 kg. weight/ton .

Problem on statisties:

(collecting data and dispersion)

First : complete the following:-

1- The statistical sample is a part of ....

2- From the means of collecting data are .... and ....

3- The arithmetic mean is one of measure of .... while' the range is one measures of ....

4- The difference between the greatest value and the smallest value of a set of data is ....

5- The arithmetic mean of a set of values of individuals equals ......

6- The positive square root of the squares deviations of values from its arithmetic mean is called ....

7- The range of the set of values 5 , 14 , 4 , 21, 16 and 12 is ......

8- The standard deviation of the set of the values 3 , 12 , 17, 28 and 30 equals ....

9- The difference between the greatest individual and the smallest individual of a set of values is called.........

10- If 78 is the greatest individual of a set of individuals and its range is 39 then the smallest individual of this set equals ....

Second : Choose the correct answer from those given:-

1- Selecting a sample of layers of a statistical society is called .... sample.

a) random b) class (payer) c) deliberate d) bunch

2- The difference between the greatest value and the smallest value of a set of individuals is called ....

a) the range
b) the arithmetic mean
c) the median
d) the standard deviation

3- The range of the set of the values 7 , 3 , 6 , 9 and 5 equals.

a) 3 b) 4 c) 6 d) 12

4- The arithmetic mean of the set of the values 7 , 3 , 6 , 9 and 5 equals..........

a) 3 b) 4 c) 6 d) 12

5- If $\sum(x - \bar{x})^2 = 36$ for a set of values whose number is 9 then $\sigma = \dots$

a) 2 b) 4 c) 18 d) 27

1- Calculate the arithmetic mean and the standard deviation of the set of values: 16 , 32 , 5 , 20 and 27

2- Calculate the standard deviation for the values:3 ,12,17 , 28 , 30.

3- Calculate the mean and standard deviation to the following data 12 ,13,16 , 18 and 21.

4- Calculate the arithmetic mean and the standard deviation of the set of values 73 , 54 , 62 , 71 , 60

5- The following table represents the number of children of 100 families in a city.

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate each of the arithmetic mean and standard deviation.

6- The following table represents the number of children of 26 families in a city.

Number of children	0	1	2	3	4	5	Total
Number of families	9	1	6	3	5	2	26

Calculate the standard deviation.

7- Find the arithmetic mean and standard deviation of the following data:

The set	0-	2-	4-	6-	8-
The frequency	5	9	15	15	6

Part Two

Model Answers

Complete each of the following:-

1) a : b = 4 : 3

2) $\frac{3}{4} = \frac{c}{8} \Rightarrow \quad c = \frac{8 \times 3}{4} = 6$

3) Proportional mean = $\pm \sqrt{1^{st} \times 3^{rd}}$

$= \pm \sqrt{3ab^2 \times 27a^3b^2} =$

$= \pm \sqrt{81a^2 \times b^4} = \pm 9a^2b^2$

4) $\frac{3x}{5y} = \frac{9}{25}$

5) $\frac{9}{25} = \frac{2x}{y^2}$

$\frac{9}{y2} = 4x^2 \Rightarrow x^2y^2 = \frac{9}{4}$

$xy = \pm \frac{3}{2}$

6) $4x^2 - 12xy + 9y^2 = 0$

$(2x - 3y)^2 = 0 \quad \Rightarrow 2x - 3y = 0$

$2x = 3y \quad \Rightarrow \frac{x}{y} = \frac{3}{2}$

7) $\frac{a}{b} = \frac{2}{3} \quad , \frac{a}{c} = \frac{3}{5}$

a : b : c
2 : 3 :
3 :   : 5

6 : 9 : 10

8) a = 7m , b= 2m → $\frac{7m-2m}{7m+2m} = \frac{5m}{9m} = \frac{5}{9}$

9) $\frac{x+y}{11} = \frac{2y+z}{14}$

10) $\frac{1}{x} = \frac{x}{9} = \frac{9}{y}$

$x^2 = 9 \Longrightarrow x = \pm \sqrt{9} = \pm 3$

$\Longrightarrow yx = 81$

$y = \frac{81}{\pm 3} = \pm 27$

2- Choose: -

1) d	2) c	3) a
4) b	5) b	6) d
7) a	8) a	9) d
10) c		

Steps for 7 , 9

7) $\frac{1+x}{3+x} = \frac{3+x}{7+x} = \frac{7+x}{15+x}$

$(3 + x)(3 + x) = (1 + x)(7 + x)$

$x^2 + 6x + 9 = x^2 + 8x + 7$

$8x - 6x + 9 - 7 \rightarrow 2x = 2$

$x = 1$

9) $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$

$z = 4 \times 2 - 2 \times 3$

$z = 8 - 6 = 2$

3-

a) $\frac{8}{x} = \frac{4}{5} \rightarrow x = 10$

b) $\frac{11}{3} = \frac{x}{6} \rightarrow x = 22$

c) $\frac{6}{24} = \frac{1}{x} \rightarrow x = 4$

4- a)

x : y : z
3 : 5 : 5
4 : 4 : 7
12 : 20 : 35

b)

x : y : z
4 : 5 : 4
3 : 3 : 7
12 : 15 : 28

5- a) $\frac{a+b}{a} = \frac{2m+5m}{2m} = \frac{7}{2}$

b) $\frac{a}{b-a} = \frac{2m}{5m-2m} = \frac{2}{3}$

c) $\frac{b-a}{b+a} = \frac{7m-2m}{7m+2m} = \frac{5}{9}$

d) $\frac{7a-2b}{3a+2b} = \frac{14m-10m}{6m+10m} = \frac{1}{4}$

6- $\frac{2a-b}{m} = \frac{2a-b}{4-3} \Rightarrow m = 1$

7- $\frac{a}{b-a} = \frac{c}{d-c}$ $\quad \therefore ad - ac = bc - ac$

$\therefore ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad \therefore$ a,b,c,d are prop.

8- $\frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore a = cm^2 \quad , b = cm$

L.H.S. $= \frac{a^2}{b^2} = \frac{c^2m^4}{c^2m^2} = m^2$ ___ (1)

M.H.S. $= \frac{b^2}{c^2} = \frac{c^2m^2}{c^2} = m^2$ ___ (2)

R.H.S. $= \frac{a}{c} = \frac{cm^2}{c} = m^2$ ___ (3)

9- x = 3k , y = 4k, z=5k

a) $\frac{2y-z}{3x-2y+z} = \frac{8k-5k}{9k-8k+5k} = \frac{3k}{6k} = \frac{1}{2}$

b) $\sqrt{3x^2+3y^2+z^2}$ L.H.S.

$= \sqrt{3(3k)^2+3(4k)^2+(5k)^2}$

$= \sqrt{27k^2+48k^2+25k^2}$

$= \sqrt{100k^2} = 10k$

2x + y = 6k + 4k = 10k R.H.S.

L.H.S. = R. H. S.

10- $\frac{a-1}{a+1} = \frac{b-2}{b+2}$

(a – 1) (b + 2) = (a + 1)(b - 2)

ab + 2a – b2 = ab – 2a + b2

2a – b = -2a+b

2a + 2a = b + b

$4a = 2b \rightarrow \frac{a}{b} = \frac{2}{4} = \frac{1}{2}$

a = m , b = 2m

L.H.S. $= \frac{a+b}{a+b-3} = \frac{m+2m}{m+2m-3} = \frac{3m}{3m-3}$

$= \frac{3m}{3(m-1)} = \frac{m}{m-1}$ ______ (1)

R.H.S. $= \frac{3a}{5a-b-3} = \frac{3m}{5m-2m-3} = \frac{3m}{3m-3}$

$= \frac{3m}{3(m-1)} = \frac{m}{m-1}$ ______ (2)

From (1) and (2) L.H.S. = R. H. S.

11- $\frac{a}{b} = \frac{1}{3}$, $\frac{a}{c} = \frac{1}{9}$, a + b +c = 26

a = m , b = 3m , c = 9m

m + 3m + 9m = 26 → 13m = 26

m=2 → a = 2 , b=6 , c=18

12- $\frac{x}{y} = \frac{z}{\ell} = m$

x = ym , z= ℓ m

L.H.S. = $\left(\frac{x+y}{z+\ell}\right)^2 = \left(\frac{ym+y}{\ell m+\ell}\right)^2 = \left(\frac{y(m+1)}{\ell(m+1)}\right)^2$

$= \left(\frac{y}{\ell}\right)^2 = \frac{y^2}{\ell^2}$ ______ (1)

R.H.S. = $\frac{2x^2-3y^2}{2z^2-3\ell^2} = \frac{2y^2\ m^2-3y^2}{2\ell^2\ m^2-3\ell^2}$

$= \frac{y^2\ (2m^2-3)}{\ell^2\ (2m^2-3)} = \frac{y^2}{\ell^2}$ ______ (2)

From (1) and (2) L.H.S. = R.H.S.

b) L.H.S. = $\sqrt[3]{\frac{5x^3-3z^3}{5y^3-3\ell^3}}$ =

$\sqrt[3]{\frac{5y^3\ m^3-3\ell^3 m^3}{5y^3-3\ell^3}} = \sqrt[3]{\frac{m^3(5y^3-3\ell^3)}{5y^3-3\ell^3}}$

$= \sqrt[3]{m^3} = m$ ______ (1)

R.H.S. = $\frac{x+z}{y+\ell} = \frac{ym+\ell m}{y+\ell} = \frac{m(y+\ell)}{(y+\ell)} = m$ ______ (2)

From (1) , (2) we get L.H.S. = R.H.S

13- 1st – 2nd + 3rd = One of the rations

$\frac{x+y-y-z+z+x}{\ell+m-m-n+n+\ell} = \frac{2x}{2\ell} = \frac{x}{\ell}$

= one of the ratios (1)

2nd – 3rd = One of the ratios

$\frac{y+z-z-x}{m+n-n-\ell} = \frac{y-x}{m-\ell}$ = one of the ratios (2)

From (1) and (2) we get

$\frac{x}{\ell} = \frac{y-x}{m-\ell}$

14- 2x 1st + 2nd

$\frac{2x+y}{4a+2b+2b-c} = \frac{2x+y}{4a+4b-c} =$

One of the given ratios. ______ (1)

2x 1st + 2x 2nd + 3rd

$\frac{2x+2y+z}{4a+2b+4b-2c+2c-a}$ = one of the given ratios

$= \frac{2x+2y+z}{3a+6b}$ = one of ratios ______ (2)

From (1) and (2) L.H.S. = R. H. S.

15- By adding 1st + 2nd + 3rd

$\frac{x+y+y+z+z+x}{7+5+8} = \frac{2x+2y+2z}{20}$

$= \frac{2(x+y+z)}{20} = \frac{x+y+z}{10}$ = one of the given ratios ______ (1)

x – z 1st – 2nd

$\frac{x+y-y-z}{7-5} = \frac{x-z}{2}$ = one of the given ratios ______ (2)

From (1) and (2) $\frac{x+y+z}{10} = \frac{x-z}{2} =$

$\frac{x+y+z}{x-z} = \frac{10}{2} = 5$

16- $\frac{7+x}{9+x} = \frac{12+x}{15+x}$

$(x + 15)(x + 7) = (x + 12)(x + 9)$

$x^2 + 7x + 15x + 105 = x^2 + 9x + 12x + 108$

$22x + 105 = 21x + 108 +$

$22x - 21x = 108 - 105$

$x = 3$

17- let the + ve. integers are x , y

$\frac{x}{y} = \frac{3}{7} = x = 3m , y=7m$

$\frac{3m-5}{7m-5} = \frac{1}{3}$

$9m-15 = 7m - 5$

$9m - 7m = 15 - 5$

$2m = 10 \Rightarrow m = 5$

18- let the no. is x

$\frac{x^2+7}{x^2+11} = \frac{4}{5}$

$5x^2 + 35 = 4x^2 + 44$

$5x^2 - 4x^2 = 44 - 35$

$x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$

Complete the following:

1) x

2) $\frac{1}{x}$

3) $\frac{y_1}{y_2}$

4) m = 4

5) $\frac{1}{2}x$

6) $\frac{1}{x}$

7) x

8) x

9) $\frac{y_2}{y_1}$

10) 2

Choose

1) d

2) c

3) c

4) d

5) d

3-

x	y
3	20
4	15
5	12
6	10

- The first table represents an inverse variation because 3x20= 5x12 = 4x15 = 6x10=60 ∴ xy = m
- The 2nd table does not represent a direct or inverse variation because 2x9 = 18 , 4x18 = 72 or $\frac{4}{18} \neq \frac{6}{54}$
- The third table represent a direct variation because $\frac{4}{5} = \frac{18}{10} = \frac{27}{15} = \frac{45}{25} \therefore \frac{y}{x} = m$
- The fourth table does not represent a direct variation or inverse because 3x6 ≠ -18x1 or $\frac{6}{3} \neq \frac{-9}{-2}$

x	y
5	9
10	18
15	27
25	45

4- $y \propto x \; \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\frac{10}{20} = \frac{7}{x}$

$x = 7 \times 20 \div 10 = 14$

5- $y \propto \frac{1}{x}$ $\quad \frac{y_1}{y_2} = \frac{x_2}{x_1}$

$\frac{10}{y} = \frac{5}{3}$

$y = 3 \times 10 \div 5 = 6$

6- $y = mx \rightarrow 20 = 7m$

$m = \frac{20}{7} \quad \rightarrow \quad y = \frac{20}{7}x$

$\frac{y_1}{y_2} = \frac{x_1}{x_2} \quad \rightarrow \quad \frac{20}{y} = \frac{7}{14}$

$y = 14 \times 20 \div 7 = 40$

7- $\because y \propto \frac{1}{x}$

$\therefore \frac{y_1}{y_2} = \frac{x_2}{x_1}$

$\frac{\frac{4}{7}}{y} = \frac{3\frac{1}{5}}{2\frac{4}{5}}$

$y = 2\frac{4}{5} \times \frac{4}{7} \div 3\frac{1}{5} = \frac{14}{5} \times \frac{4}{7} \div \frac{16}{5} =$

$y = \frac{14}{5} \times \frac{4}{7} \times \frac{5}{16} = \frac{1}{2}$

8- $y = 3 + a$, $a \propto \frac{1}{x}$

at $y = 5 \rightarrow 5 = 3 + a_1$ $\quad \left\{ \begin{array}{l} Y_1 = 5 \\ X_1 = 1 \end{array} \right.$

$\rightarrow a_1 = 2$

$\frac{a_1}{a_2} = \frac{x_2}{x_1}$ $\quad \left\{ \begin{array}{l} Y_2 = ? \\ X_2 = 2 \end{array} \right.$

$\frac{2}{a_2} = \frac{2}{1} \Rightarrow a_2 = 1 \times 2 \div 2 = 1$

at $a_2 = 1 \Rightarrow \quad y_2 = 3 + 1 = 4$

9- $y = a + 7$ and $a \propto \frac{1}{x^2}$ $\quad \left\{ \begin{array}{l} a = 18 \\ x_1 = \frac{2}{3} \end{array} \right.$

$a = \frac{m}{x^2} \rightarrow m = ax^2$

$m = 18 \times \frac{4}{9} = 8$ $\quad \left\{ \begin{array}{l} y = ? \\ x = 6 \end{array} \right.$

$a = \frac{8}{x^2}$

$a = \frac{8}{(6)^2} = 8 \div 36 = \frac{2}{9}$

at $a = \frac{2}{9} \rightarrow y = \frac{2}{9} + 7 = 7\frac{2}{9}$

no. (5) $y \propto \frac{1}{x}$

$\therefore y = \frac{m}{x} \quad \therefore 10 = \frac{m}{3} \quad \therefore m = 30$

$\boxed{\therefore y = \frac{30}{x}}$

$\therefore y = \frac{30}{5} \quad \boxed{\therefore y = 6}$

Another solution for no (8)

No. 8 $\quad y = a + 3$

$\because a \propto \frac{1}{x} \quad \therefore a = \frac{m}{x}$

$\therefore y = \frac{m}{x} + 3$

$\therefore 5 = m + 3 \quad \therefore m = 2$

$\boxed{\therefore y = \frac{2}{x} + 3}$

$\therefore y = \frac{2}{2} + 3 \quad \boxed{\therefore y = 4}$

Another solution for no (9)

No. 9 $y = a + 7$

$\because a \propto \frac{1}{x^2}$ $\therefore a = \frac{m}{x^2}$

$\therefore 18 = \frac{9m}{4}$ $\therefore m = \frac{4 \times 18}{9} = 8$

$\therefore y = \frac{8}{x^2} + 7$

$\therefore y = \frac{8}{36} + 7 = \frac{65}{9}$

10- $\frac{21x-y}{7x-z} = \frac{y}{z}$

$z(21x - y) = y(7x - z)$

$21xz - yz = 7xy - yz$

$21xz = 7xy$

$3z = y \rightarrow \therefore y = mz$

$\therefore y \propto z$

11- a) inverse variation

b) $\because y \propto \frac{1}{x}$ $\therefore m = xy = 2 \times 6 = 12$

c) $y = \frac{12}{x}$ at $x = 3$

$y = \frac{12}{3} = 4$

d) at $y = 2\frac{2}{5} = \frac{12}{5}$ $x = 12 \div \frac{12}{5} = 5$

12- $d_1 = 90$ km $t_1 = 1\frac{1}{2}$ h

$d_2 = ?$ $t_2 = 2\frac{1}{2}$ h

$d \propto t \rightarrow d = mt$

$90 = 1\frac{1}{2} m \rightarrow m = 90 \div \frac{3}{2} = 60$

$d = 60t$

$d = 60 \times 2\frac{1}{2} = 60 \times \frac{5}{2} = 150$ km.

13- $t \propto \frac{1}{d^2}$ $d_1 = 12$

$t = \frac{m}{d^2}$ $t_1 = x$

$x = \frac{m}{12^2}$ $d_2 = ?$

$m = 12x = 144x$ $t_2 = 4x$

at $t = 4x$ $4x = \frac{12x}{d^2}$

$d^2 = \frac{144}{4} = 36$

$d = \sqrt{36} = 6$

Complete each of the following

1. The point (5 , - 3) lies in ……….. quadrant
2. The point (4 , 0) lies on ……… - axis
3. If : (5 , x – 7) = (y + 1 , - 5) , then x + y = ……..
4. If : (x + 5 , 8) = (1 , 6 y + x) , then y = ……..
5. If : n (x) = 5 , n (x x Y) = 15 , then n (Y) = ……..
6. If : x x Y = { (1 , 5) , (1 , 7) , (2 , 5) , (2 , 7) , (3 , 5) , (3 , 7) } , then x = ……
7. If f is function where f : x → Y , then X is called …… and Y is called ……
8. If f is function from set x to set Y , then the range of function f ⊂ ……
9. If : f (x) = 5 x – 7 , then f (3) = ……
10. If : f (x) = 6 x , then f (2) + f (-2) = ……
11. If : f (x) = 3 x + b , f (4) = 13 , then b = ……..
12. If : f (x) = x – 6 and $\frac{1}{3}$ f (a) = - 2 , then a = …….
13. If : x = { 1 , 3 , 5 } f : X → ℝ and f (X) = 2 X + 1 , then the range of f = …….
14. Function f : ℝ → ℝ such that f (x) = 3 x represented by a straight line passes through the point (- 4 , ………)
15. The linear function f : f (x) = x + 7 is represented by a straight line cuts x - axis at the point ………
16. The linear function f : f (x) = 2 x – 1 is represented by a straight line cuts y - axis at the point ……….
17. The linear function f : f (x) = 2 – 3 X is represented by straight line cuts y – axis at point ……….
18. If the point (a , 3) lies on the straight line which represents the function f : ℝ → ℝ where f (x) = x – 5 , then a = …….
19. If f is a function where f (x) = 3 x – 1 is represented graphically by a straight line passes through the point (a , 2) then a = ……….
20. If : (2 , - 6) ∈ f: f (x) = k x + 8 , then k = ………

Choose the correct answer from those given

(1) The point $(-3, 4)$ lies in ……… quadrant			
ⓐ first	ⓑ second	ⓒ third	ⓓ fourth

(2) If : $x = \{5\}, Y = \{3\}$, then $n(x \times Y) =$ ……..			
ⓐ 15	ⓑ 8	ⓒ 2	ⓓ 1

(3) If : $x = \{5, 6, 7\}$, then $n(x^2) =$ ……..			
ⓐ 3	ⓑ 6	ⓒ 9	ⓓ 12

(4) If : $n(x)^2 = 9$, then $n(x) =$ ……..			
ⓐ 3	ⓑ 6	ⓒ 18	ⓓ 81

(5) If $x \times Y = \{(1, 3), (1, 4)\}$, then $n(x) =$……..			
ⓐ 3	ⓑ 1	ⓒ 4	ⓓ 2

(6) If: $x \{3, 5, 7\}$ and R is a relation on x, then the relation which represents a function is ……..	
ⓐ R = { (3 , 5) , (5 , 3) , (3 , 7) }	ⓑ R = { (3 , 5) , (5 , 7) }
ⓒ R = { (3 , 5) , (5 , 5) , (7 , 5) }	ⓓ R = { (3 , 3) , (3 , 5) , (3 , 7) }

(7) If R is a function from set x to set Y where $x = \{2, 5, 8\}$, $y = \{3, 5\}$ and $R = \{(2, 3), (5, 3), (X, 3)\}$, then $x =$……..			
ⓐ 2	ⓑ 3	ⓒ 5	ⓓ 8

(8) If the function f is a function from set x to set Y then the domain of the function is ……..			
ⓐ X	ⓑ Y	ⓒ X x Y	ⓓ Y x X

(9) If R is a function where $R = \{(4, 3), (5, 6), (9, 3)\}$ then the range of the function R is ………	
ⓐ { 3 , 4 , 5 , 6 , 9 }	ⓑ { 4 , 5 , 9 }
ⓒ { 3 , 6 , 9 }	ⓓ { 3 , 6 }

(10) If the point $(x, 7)$ lies on y − axis, then $5x + 1 = \dots\dots$			
ⓐ zero	ⓑ 1	ⓒ 5	ⓓ 6

(11) If : $f(x) = x^2 + 7$, then $f(3) = \dots\dots$			
ⓐ 10	ⓑ 7	ⓒ 9	ⓓ 16

(12) If : $f(x) = x^3$ then $f(2) + f(-2) = \dots\dots$			
ⓐ 16	ⓑ zero	ⓒ -7	ⓓ 4

(13) If : $f(x) = 7x - \frac{1}{2}$, then $f(\frac{1}{2}) = \dots\dots$			
ⓐ 7	ⓑ $\frac{1}{2}$	ⓒ $\frac{7}{2}$	ⓓ 3

(14) The function f, where $f(x) = 5x$ is represented graphically by a straight line passes through the point ……			
ⓐ (5 , 5)	ⓑ (0 , 0)	ⓒ (0 , 5)	ⓓ (5 , 0)

(15) If : $f(x) = 4x + b$, $f(3) = 15$, then $b = \dots\dots$			
ⓐ 156	ⓑ 3	ⓒ 4	ⓓ -3

(16) If : (m , 13) satisfies the function f where $f(x) = 3x + 4$, then $m = \dots\dots$			
ⓐ 6	ⓑ -6	ⓒ 3	ⓓ -3

(17) If : (2 , b) satisfies the function f where $f(x) = 3x - 6$ then $b = \dots\dots$			
ⓐ Zero	ⓑ 7	ⓒ 9	ⓓ 2

(18) If: $f(x) = 5x + 4$ is represented graphically by a straight line passes through the point (3 , b), then $b = \dots\dots$			
ⓐ 5	ⓑ 4	ⓒ 3	ⓓ 19

Answer the following questions

1 If : x = { 0 , 1 , 2 , 3 , 4 , 5 , 6 } and R is a relation on x where a R b means " a is twice b " for all a ∈ x , b ∈ x , a ≠ b

(1) Write R and represent it by an arrow diagram

(2) Is (0 , 0) ∈ R

(3) Is 2 R 4 ?

(4) Find x if 6 R x

---

2 If : x = { 2 , 4 , 8 } , x = { 4 , 6 , 12 , 24 } , and R is a relation from x to Y such that a R b means " b >2 a " for all a ∈ x , b ∈ Y , write R and represent it by an arrow diagram and by a cartesian diagram

---

3 If : x = { 13 , 14 , 43 , 84 } , and R is a relation on x such that a R b means " two numbers a and b have the same unit digit " for all a ∈ x , b ∈ x Write R and represent it on a cartesian diagram

---

4 If : x = { 2 , 3 , 4 , 7 } , Y = { 1 , 2 , 3 , 4 , 7 , 8 } and R is a relation from x to Y where a R b means " a – b is a prime number " for all a ∈ x , b ∈ Y Write R and represent it andr by an arrow diagram

---

5 If : x = { 0 , 1 , 2 , 3 } , Y = { - 3 , – 2 , - 1 , 0 } and R is a relation from x to Y where a R b means " a is additive inverse of b " for all a ∈ x , b ∈ Y + write R and represent it by an arrow diagram and by a cartesian diagram . Is R a function ? why ?

---

6 If : x = { 2 , 5 , 8 } , Y = { 10 , 16 , 24 , 30 } and R is a relation from x to Y where a R b means " a is a factor of b " for all a ∈ x , b ∈ Y write R and represent it by an arrow diagram. Is R a function ? why ?

---

7) If : $\mathcal{X}$ = { 1 , 3 , 4 , 5 } , Y = { 1 , 2 , 3 , 4 , 5 , 6 } and R is a relation from $\mathcal{X}$ to Y where a R b means " a + b = 7 " for all a ∈ $\mathcal{X}$, b ∈ Y , write R and represent it by an arrow diagram and by a cartesian diagram, show that R is a function ? write its domain and its range

---

8) If : $\mathcal{X}$ = { 1 , 2 , 3 } , Y = { 1 , $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ } and R is a relation from $\mathcal{X}$ to Y where a R b means " a is the multiplicative inverse of b " for all a ∈ $\mathcal{X}$, b ∈ Y , write R and represent it by an arrow diagram and by a cartesian diagram . Is R a function ? why ?

---

9) If : $\mathcal{X}$ = { 1 , 2 , 4 } , R is a relation on $\mathcal{X}$ such that "a is a multiple of b" for all a ∈ $\mathcal{X}$, b ∈ Y write R and represent it by an arrow diagram and by a cartesian diagram . Is R a function ? why ?

---

10) If : $\mathcal{X}$ = { 2 , 3 , 4 } , Y = { 3 , 4 , 5 , 6 , 7 , 8 } and f : $\mathcal{X}$ → Y where f ($\mathcal{X}$) = 9 – $\mathcal{X}$ find the images of the elements of $\mathcal{X}$ and represent it by an arrow diagram .

---

11) If : $\mathcal{X}$ = { 3 , 4 , 5 , 10 , 13 } , Y = { 4 , 5 , 7 , 8 , 9 , 19 , 25 } and R is a relation from $\mathcal{X}$ to Y such that a R b means " b = 2a – 1 " for all a ∈ $\mathcal{X}$ and b ∈ Y :

(1) Write R

(2) Represent R by a cartesian diagram

(3) Find the value of $\mathcal{X}$ if (X , 9) ∈ R

---

12) If : $\mathcal{X}$ = { 1 , 2 , 3 } , Y = { 1 , 3 , 6 , 9 , 13 } and R is a relation from $\mathcal{X}$ to Y where a R b means " a = $\frac{1}{3}$b " for all a ∈ $\mathcal{X}$, b ∈ Y , write R and show that it is a function , write its range

---

13) If : $\mathcal{X}$ = { 3 , 5 , 7 , 9 } , Y = { a : a ∈ N , 10 ≤ a < 50 } and R is a relation from $\mathcal{X}$ to Y , where R = { (3 , 15) , (5 , 25) , (7 , 35) , (9 , 45) }

(1) What is the range of R ?

(2) Write a rule of R

(14) If function f = { (1 , 3) , (2 , 5) , (3 , 7) , (4 , 9) , (5 , 11) }

(1) Write each of domain and range of f

(2) Write the rule of the function f

---

(15) If : x = { 1 , 3 , 5 } and R is a function on x where R = { (a , 3) , (b , 1) , (1 , 5) } , then find the numerical value of the expression : a + b

---

(16) Represent the following linear function graphically :

(1) F (x) = 3 x + 1

(2) F (x) = 2 – x

(3) F (x) = 5 x

(4) F (x) = - 2 x

---

(17) Graph the function f where f (x) = 4 - x^2 in the interval [-3 , 3] , from the graph determine :

(1) The coordinates of the maximum value of function .

(2) The equation of the axis of symmetry .

---

(18) Graph the function f where f (x)= x (6 – x) + 4 in the interval [- 1 , 7]

---

(19) the straight line which represents the function f : ℝ → ℝ where f (x) = 6 x – a cuts y - axis at the point (b , 3) , then find the value of a and b

---

(20) If f : ℝ → ℝ is represented by a straight line cuts y-axis at (b , 3) where f (x) = 6 x – a Find the value of 2a + 7b

Complete each of the following

1. If : $3a = 4b$, then $a : b = \dots\dots : \dots\dots$

2. If : $\frac{x}{3} = \frac{y}{5}$, than $\frac{3x}{5y} = \dots\dots\dots\dots$

3. If : $4x^2 - 12xy + 9y^2 = 0$ and $x \in \mathbb{R}, y \in \mathbb{R}, y \neq 0$, then $\frac{x}{y} = \frac{\dots\dots}{\dots\dots}$

4. If : $\frac{a}{b} = \frac{7}{2}$, then $\frac{a-b}{a+b} = \frac{\dots\dots}{\dots\dots}$

5. If : $y^2 - 6xy + 9x^2 = 0$, then $y \propto \dots\dots$

6. $\frac{x}{6} = \frac{y}{5} = \frac{z}{4} = \frac{\dots\dots\dots}{11} = \frac{2y+z}{\dots\dots\dots}$

7. If : 3 , 4 , c and 8 are proportional , then c = ……………

8. The proportional mean of $3ab^2$ and $27a^3b^2$ is ……..

9. If : 9 , $2x$, $\frac{1}{y^2}$ are proportional quantities , then $xy = \dots\dots$

10. If : 1 , x , 9 , y are in continued proportion , then $x = \dots\dots$, $y = \dots\dots$

11. If : $y = 3x$, then $y \propto \dots\dots$

12. If : $xy - 7 = 0$, then $y \propto \dots\dots$

13. If $y \propto x$ and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively , then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$

14. If $y \propto \frac{1}{x}$ and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively , then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$

15. If $y \propto x$ and $y = 2$ when $x = 4$, then $y = \dots\dots x$

16. If y varies inversely as x and $y = 2$ when $x = \frac{1}{2}$, then $y \frac{\dots\dots}{x}$

17. If $y \propto x$ and $y = 1$ when $x = 4$, then $y = \dots\dots$ when $x = 8$

18. If : $x^2y^2 - 4xy + 4 = 0$, then $y \propto \dots\dots\dots$

Choose the correct answer from those given

(1) If : a , b , 2 and 3 are proportional , then $\frac{a}{b}$ = ……..			
ⓐ $\frac{2}{3}$	ⓑ $\frac{3}{2}$	ⓒ $\frac{3}{4}$	ⓓ $\frac{4}{3}$

(2) If :$\frac{x}{y} = \frac{z}{\ell}$ which of the following is true ?			
ⓐ $\frac{x}{\ell} = \frac{y}{z}$	ⓑ $\frac{x}{z} = \frac{\ell}{y}$	ⓒ $\frac{x}{y} = \frac{\ell}{z}$	ⓓ $\frac{x}{z} = \frac{y}{\ell}$

(3) The second proportion of the quantities 12 ab^2 , …… , 21 ab , 14 b^2 is …..			
ⓐ 8 ab^2	ⓑ 8 b^3	ⓒ 24 ab	ⓓ 24 b^2

(4) The third proportion of the two numbers 3 and 6 is …..			
ⓐ $\frac{1}{2}$	ⓑ 2	ⓒ 3	ⓓ 12

(5) If : 2 , 6 , x + 15 are proportional , then x = …………			
ⓐ 1	ⓑ 2	ⓒ 3	ⓓ 4

(6) If : $\frac{9}{a^2} = \frac{4}{b^2}$ (where a ≠ 0 and b ≠ 0) , then $\frac{a}{b}$ = ……….			
ⓐ $\frac{2}{3}$	ⓑ $\pm\frac{3}{2}$	ⓒ $\pm\frac{2}{3}$	ⓓ $\pm\frac{4}{9}$

(7) If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a}$ equals ………..			
ⓐ $\frac{1}{5}$	ⓑ $\frac{1}{3}$	ⓒ $\frac{2}{5}$	ⓓ $\frac{3}{5}$

(8) If $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$, then z = ……….			
ⓐ -2	ⓑ $-\frac{1}{2}$	ⓒ $\frac{1}{2}$	ⓓ 2

(9) If : $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ m (where m ∈ $\mathbb{R}^y$) , then $\frac{a\ c\ e}{b\ d\ f}$ equals …..			
ⓐ m	ⓑ 3m	ⓒ m^3	ⓓ 3 m^3

(10) The number which if we added to each of the numbers 1 , 3 , 7 , 15 respectively to be in continued is ……….

ⓐ 1	ⓑ 2	ⓒ 3	ⓓ 4

(11) The relation which represents direct variation between the two variable x and y is ……………

ⓐ $x\,y = 7$	ⓑ $y = x + 2$	ⓒ $\frac{x}{3} = \frac{4}{y}$	ⓓ $\frac{x}{5} = \frac{y}{2}$

(12) If $y \propto x$ and $x = 1$ at $y = 4$, then the constant of the variation equals …….

ⓐ 1	ⓑ -4	ⓒ $\frac{1}{4}$	ⓓ $-\frac{1}{4}$

(13) If y varies inversely as x and if $x = \sqrt{3}$ as $y = \frac{2}{\sqrt{3}}$, then the constant of variation equals………

ⓐ $\frac{1}{2}$	ⓑ $\frac{2}{3}$	ⓒ 2	ⓓ 6

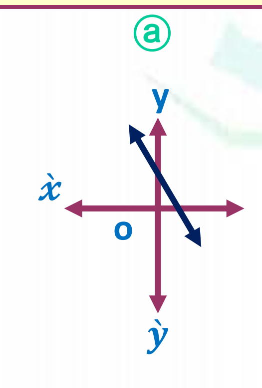
(14) If : $y - x = \frac{1}{x} - \frac{1}{y}$ *where* $x \neq y \neq 0$, *then* … … ….

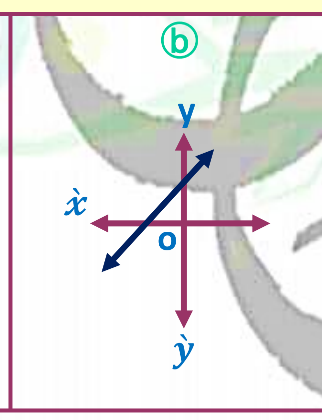
ⓐ $y \propto x + 1$	ⓑ $y \propto x$	ⓒ $y \propto \frac{1}{x}$	ⓓ $y \propto \frac{1}{x^2}$

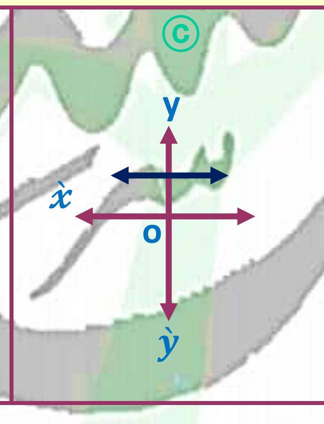
(15) If some of the total cost (y) for a certain trip is constant (a) and the other changes with the number of participants (x), which of the following relations is correct ?

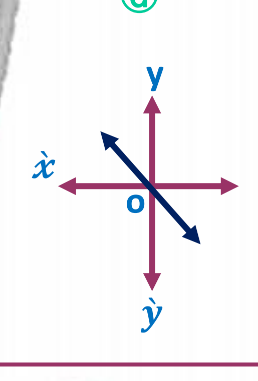
ⓐ $y = a\,x$	ⓑ $y = \frac{a}{x}$
ⓒ $y = a + \frac{m}{x}$, (m is a constant $\neq 0$)	ⓓ $y = a + m\,x$, (m is a constant $\neq 0$)

(16) The graph which represent the direct variation between x and y is …….

ⓐ


ⓑ


ⓒ


ⓓ


Essay questions

1. If the following sets of numbers are proportional, then find the values of x

(1) $8 , x , 4 , 5$ (2) $11 , 3 , x , 6$ (3) $6 , 24 , 1 , x$

2. Find : $x : y : z$ in each of the following

(1) $\frac{x}{y} = \frac{3}{5}$ and $\frac{y}{z} = \frac{4}{7}$ (2) $\frac{x}{y} = \frac{4}{5}$ and $\frac{x}{z} = \frac{3}{7}$

3. If : $\frac{a}{b} = \frac{2}{5}$, then find the value of each of the following rations:

(1) $\frac{a+b}{b}$ (2) $\frac{a}{b-a}$ (3) $\frac{b-a}{b+a}$ (4) $\frac{7a-2b}{3a+2b}$

4. If : $\frac{a}{b-a} = \frac{c}{d-c}$, then prove that : a , b , c and d are proportional

5. If b is the middle proportional between a and c, then prove that:

(1) $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$ (2) $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = b^2$

6. If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that :

(1) $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$ (2) $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

7. If : a -1 , a + 1 , b – 2 , b +2 are in proportion , then find $\frac{a}{b}$, then prove that : $\frac{a+b}{a+b-3} = \frac{3a}{5a-b-3}$

(8) If : $\frac{a}{b} = \frac{1}{3}$, $\frac{a}{c} = \frac{1}{9}$ *and* $a + b + c = 26$ *, then find each of a , b and c*

(9) If x , y , z , ℓ are proportional quantities , then prove that:

(1) $\left(\frac{x-y}{z+\ell}\right)^2 = \frac{2x^2 - 3y^2}{2z^2 - 3\ell^2}$ (2) $\sqrt[3]{\frac{5x^3 - 3z^3}{5y^3 - 3\ell^3}} = \frac{x+z}{y+\ell}$

(10) If : $\frac{x+y}{\ell+m} = \frac{y+z}{m+n} = \frac{z+x}{n+\ell}$, then prove that : $\frac{x}{\ell} = \frac{y-x}{m-\ell}$

(11) If : $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$ = , *then prove that* : $\frac{2x+y}{4a+4b+c} = \frac{2x+2y+z}{3a+6b}$

(12) If : $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$, then prove that : $\frac{x+y+z}{x-z} = 5$

(13) Find the number that should be added to each of the numbers: 7, 9, 12 , 15 to be proportional .

(14) Two positive integer numbers, the ratio between them is 3 : 7 and if we subtract 5 from each of them the ratio becomes 1 : 3 , find the two numbers.

(15) Find the positive number that if we add its square to each term of the ratio 7: 11 it becomes 4: 5

(16) If y varies directly as x and y = 10 when x = 7, find x when y = 20

(17) If y varies inversely as x and y = 10 when x = 3 , find y when x = 5

(18) If $y \propto x$ and y = 20 when x = 7 find the relation between x and y then find the value of y when x = 14

(19) If $y \propto \frac{1}{x}$ and $x = 2\frac{4}{5}$ when $y = \frac{4}{7}$, then find the relation between x and y then find also the value of y when $x = 3\frac{1}{5}$

(20) If y = 3 + a and $a \propto \frac{1}{x}$ if y = 5 when x = 1 , then find the relation between x and y and find y when x = 2

(21) Let y = a + 7 and $a \propto \frac{1}{x^2}$ if a = 18 when $x = \frac{2}{3}$ find the relation between y and x then deduce the value of y when x = 6

(22) If , $\frac{12x-y}{7x-z} = \frac{y}{z}$ then prove that : $y \propto z$

(23) From the data of the following table answer the following questions

x	2	4	6
y	6	3	2

(1) Identify the kind variation whether it is direct or inverse

(2) Find the constant of variation

(3) Find the value of y when x = 3 (4) Find the value of x when $y = 2\frac{2}{5}$

Complete the following

1. The resources of collecting data are…………..and………………..

2. The personal interview is a......... resource of collecting data.

3. The data of the students that are registered in students affair is a………….. resource of collection data.

4. Central agency for public mobilization and statistics is a……… resource of collecting data.

5. Direct observing is a……..... resource of collecting data.

6. The suitable method for examining blood of a patient is a………..

7. The suitable method for checking the production of a factory is…………

8. The suitable method to know the population is.........

9. The suitable method to know the ratio of absence in one of the schools is……

10. If the society is divided into illiterates and literates, carries of mediate, intermediate and high qualifications, the choosen sample for making a research is called…………………..

11. Dispersion measurements are………and………………

12. The simplest measure of the dispersion is.......

13. The difference between the greatest value and the smallest value in a set of values is called…………..

14. The positive square root of the average of squares of deviations of the values from their mean is called……….....

15. If the standard deviation equals zero, then...............

16. The dispersion to any set equally values equals…………

17. The mean of the set of the values: 7,5,9, 11 and 3 is……….

(18) The range of the set of the values: 6,5,9,4 and 12 is................

(19) If the standard deviation for nine of the values is 3, then : $\Sigma\left(x-\overline{x}\right)^2$ for these values is …………

Choose the correct from those given :

(1) ..... is a secondary resource of collecting data.	
ⓐ Personal interview	ⓑ Questionnaires
ⓒ Data base of the employees	ⓓ Observing and measuring

(2) …………is a primary resource of collecting data.	
ⓐ Central agency for statistics	ⓑ Questionnaires
ⓒ Data of the school pupils in the previous year	
ⓓ Data of the employees in one of the companies	

(3) The method of mass population is suitable for...........
ⓐ searching the formation of the sand of the Western Desert.
ⓑ examining the sweetness of water for one of the wells.
ⓒ finding out the ratio of finding a metal in one of the mines.
ⓓ getting the number of the students who had the full mark in maths exam in a class.

(4) Choosing a sample from the society's layers in statistics is called. ....sample.			
ⓐ biased	ⓑ layer	ⓒ international	ⓓ cluster

(5) The mean of the values: 3,5,7 and 9 equals			
ⓐ 9	ⓑ 3	ⓒ 8	ⓓ 6

(6) The range of the set of values: 8,3, 10,5 and 1.2. ....

ⓐ 3	ⓑ 9	ⓒ 10	ⓓ 4

(7) The most repeated value in a set of values represents.................

ⓐ the median	ⓑ the range.	ⓒ the mode.	ⓓ the mean.

(8) If the mean of numbers: 3k-3, 3k-1, 2k + 1, 2k + 3 and 2k + 5 is 13, then k = ...

ⓐ -5	ⓑ 10	ⓒ 5	ⓓ $\frac{1}{5}$

(9) $\frac{\text{sum of values}}{\text{number of these values}}$ = ...

ⓐ range	ⓑ standard deviation	ⓒ Mean	ⓓ mode

(10) If $\Sigma(x - \overline{x})^2 = 36$ of a set of values and the number of these values = 9, then the standard deviation =

ⓐ 2	ⓑ 18	ⓒ 27	ⓓ 4

Third Essay questions

1 The following table shows the frequency distribution of the number of students who won in an art competition from a school having 20 classes

Number of students	0	1	2	3	4	5	total
Number of classes	1	3	5	6	3	2	20

Find the mean and the standare deviation of the number of students

" 2.6"

2 The following table represents the frequency distribution of sets of temperature degrees in some of world cities

Sets of temperature degrees	5 -	15 -	25 -	35 -	45 -
frequency	7	9	11	15	8

Find the mean and the standard deviation of the temperature degrees.

" 31.6 , 12.9 "

---

3 Calculate the mean and standard deviation of the following data :

(1) 65 , 61 , 70 , 54 , 70 , 76 , 70 " 68 , 4.6 "

(2) 23 , 12 , 17 , 13 , 15 , 16 , 8 , 9 , 37 , 10 " 16 , 8.2 "



Choose the correct answer:

(1) If (a+5 , 3) = (8 , b−1) then $\sqrt{a^2 + b^2}$ = ...

a 7 b 3 c 9 d 5

(2) If (X^5 , Y+1) = (32 , $\sqrt[3]{27}$), then X − Y = ...

a 0 b 4 c 2 d 5

(3) If n(X^2) = 9, then n(X) = ...

a 3 b ±3 c 9 d ±9

(4) If n(Y) = 3 and n(X×Y) = 12, then n(X^2) = ...

a 4 b 16 c 9 d 2

(5) If n(X^2) = 9 and n(X×Y) = 6, then n(Y^2) = ...

a 3 b 2 c 4 d 8

(6) If X = {2} and Y = {3}, then X×Y = ...

a 6 b {6} c (2,3) d {(2,3)}

(7) If X = {5}, then n(X^2) = ...

a 1 b 25 c 10 d 5

(8) If X = {1,2} and Y = {3,4}, then (3,4) ∈ ....

a X×Y b Y×X c X^2 d Y^2

(9) If n(X) = 2 and Y = {1,2}, then n(X×Y) = ...

a 4 b 3 c 5 d 6

(10) For any two sets A and B, then the set $\{(x,y): x \in A,\ y \in B\}$ refers to …………

ⓐ $n(A\times B)$ ⓑ $A\times B$ ⓒ $n(B\times A)$ ⓓ $B\times A$

(11) If $X = \{3,4\}$, then $n(X\times\varnothing) =$ ………

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ $\varnothing$

(12) If $n(X) = k-2$, $n(Y) = k+2$ and $n(X\times Y) = 5$, then $k =$ ………

ⓐ 3 ⓑ -3 ⓒ ± 3 ⓓ 0

(13) If $\{2\} \times \{x,y\} = \{(2,4)\ ,\ (2,3)\}$, then $x-y =$ ………

ⓐ 1 ⓑ -1 ⓒ ± 1 ⓓ 0

(14) If the point $(a,5) \in$ Y-axis, then $a =$ ………

ⓐ 0 ⓑ 5 ⓒ -5 ⓓ 25

(15) If the point $(5,b-7) \in$ X-axis, then $b =$ ………

ⓐ 2 ⓑ 5 ⓒ 7 ⓓ 12

(16) If $b < 3$, then the point $(5,b-3)$ lies in the ………… quadrant.

ⓐ first ⓑ second ⓒ third ⓓ fourth

(17) If (a,b) lies in the third quadrant, then a b ………… zero

ⓐ = ⓑ $<$ ⓒ $>$ ⓓ $\leq$

(18) If $(|x|,4) = (3,y^2)$ and (x,y) lies in 2nd quadrant, then $x+y=$……

ⓐ 7 ⓑ 1 ⓒ -1 ⓓ -7

(19) If $(x-2,x-4)$ lies in 4th quadrant, then $x =$ ………

ⓐ 0 ⓑ 2 ⓒ 3 ⓓ 4

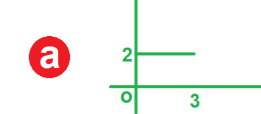
(20) If (k^2-4,k) lies on the negative direction of Y-axis, then $k=$……

ⓐ 2 ⓑ ± 2 ⓒ -2 ⓓ 0

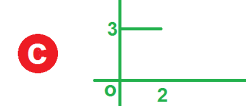
(21) If X×Y = {(1,2) , (1,3) , (1,4)}, then $n(X^2)$ = ...

ⓐ 0 ⓑ 1 ⓒ {(1,1)} ⓓ 9

(22) {3} × [0,2] is represented by the figure ....

ⓐ 

ⓑ 

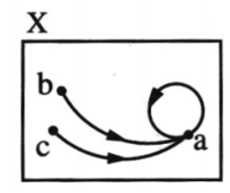
ⓒ 

ⓓ 2 o 3

(23) If R = {(1,3) , (2,5) , (4,3)} represent a function, then its domain = ...

ⓐ {1,2,4} ⓑ {3,5,4} ⓒ Z ⓓ N

(24) The opposite figure represent the arrow diagram of a function on X.
The range = ...



ⓐ {a} ⓑ {a,b} ⓒ {a,b,c} ⓓ {b,c}

(25) The set of images of each element of the domain of the function is called the ....

ⓐ domain ⓑ codomain ⓒ range ⓓ rule

(26) If the function f :X→Y, then the range ⊂ ....

ⓐ X×Y ⓑ X ⓒ Y ⓓ Y×X

(27) The function $f(x) = x^5-3x^4+1$ is of .... degree.

ⓐ 4^{th} ⓑ 9^{th} ⓒ 5^{th} ⓓ 2^{nd}

(28) The function $f(x) = x(x-x^2)$ is a polynomial of .... degree.

ⓐ 1^{st} ⓑ 2^{nd} ⓒ 3^{rd} ⓓ 4^{th}

(29) The function $f(x) = x^2-(x^2-3x)$ is a polynomial of .... degree.

ⓐ 1^{st} ⓑ 2^{nd} ⓒ 3^{rd} ⓓ 4^{th}

(30) If $a = 0$ and $b \neq 0$, then the polynomial $f(x) = ax^2+bx+c$ is of ……… degree.

a) 1st b) 2nd c) 3rd d) 4th

(31) If $f(x) = x^2-1$, then $f(1) =$ ………

a) 0 b) 2 c) -2 d) 1

(32) If $f(x) = x^2 - \sqrt{2}x$, then $f(\sqrt{2}) =$ ………

a) 4 b) 2 c) 6 d) 0

(33) If $f(x) = kx + 8$ and $f(2) = 0$, then $k =$ ………

a) 8 b) 6 c) 4 d) -4

(34) If $f(x) = nx^2 + 3x^n - 3$, the set of all possible values of n that makes the function is of 2nd degree is ……………

a) {2,3} b) {1,-1} c) {0,1,2} d) {2,1}

(35) If $(a,a) \in f$ where $f(x) = 2x + 3$, then $a =$ ……

a) 3 b) -3 c) 0 d) 1

(36) If $X = \{1,2,3\} \rightarrow f(x) = x^2 - 1$, then $f(4) =$ ……

a) 15 b) 17 c) 3 d) undefined

(37) If the curve that represents the function $f(x) = x^2 + c$ passes through the point (0,2), then $c =$ ……

a) 3 b) 2 c) -3 d) 1

(38) The vertex of the curve that represents the function $f(x) = 2x^2 - 4x + 5$ is ………………

a) (1,3) b) (3,1) c) (-1,3) d) (3,-1)

(39) If $f(x) = 5$, then $f(-3) =$ …………

a) 5 b) -5 c) -3 d) -15



(40) If $f(x) = 2$, then $f(3) - f(1) =$ ....

ⓐ 0 ⓑ $f(2)$ ⓒ 2 ⓓ 10

(41) If $f(x) = 4$, then $f(4) \div f(10) =$ ....

ⓐ 4 ⓑ $\frac{2}{5}$ ⓒ 1 ⓓ 10

(42) If $f(2x) = 4$, then $f(-x) =$ ....

ⓐ -2 ⓑ -4 ⓒ 4 ⓓ 2

(43) $f(x) = 3x$ is represented by a straight line passes through the point ....

ⓐ (3,3) ⓑ (3,0) ⓒ (0,0) ⓓ (0,3)

(44) If the straight line that represents the function $f(x) = 2x - a$ passes through the origin, then a = ...

ⓐ -3 ⓑ 2 ⓒ 0 ⓓ 3

(45) If $(a,4) \in f$ where $f(x) = 2x + b$, then $6a + 3b =$ ....

ⓐ 12 ⓑ 9 ⓒ 6 ⓓ 3

(46) If $f(x) = x^2$ and $x \in [-2,2]$, then $f(x) \in$ ....

ⓐ [0,4] ⓑ]0,4[ⓒ [0,1] ⓓ [-4,4]

(47) If (x,7) is located on Y-axis, then $5x + 1 =$ ....

ⓐ 0 ⓑ 1 ⓒ 5 ⓓ 6

(48) If (a,3) lies on the straight line that represents $f(x) = 2x - 5$, then a = ...

ⓐ 1 ⓑ 2 ⓒ -2 ⓓ 4

(49) If $f(x) = 3x + b$ and $f(4) = 13$, then b = ...

ⓐ 1 ⓑ 2 ⓒ 0 ⓓ 3

(50) If $f(x) = x - 6$ and $\frac{1}{3} f(a) = -2$, then a = ...

ⓐ 1 ⓑ 0 ⓒ 2 ⓓ 6

(51) The ordered pair (x,y) where $x > 0$ and $y < 0$ is located in the .... quadrant.

ⓐ 1^{st} ⓑ 2^{nd} ⓒ 3^{rd} ⓓ 4^{th}

(52) If $2x = 7y$, then $\left(\frac{x}{y}\right)^{-1}$ = ...

ⓐ $\frac{2}{7}$ ⓑ $\frac{7}{2}$ ⓒ $\frac{49}{4}$ ⓓ $\frac{4}{49}$

(53) If a,b,2,3 are proportional, then $\frac{b}{a}$ = ...

ⓐ $\frac{3}{2}$ ⓑ $\frac{2}{3}$ ⓒ 3 ⓓ 2

(54) If a,1,b,2 are proportional, then $\frac{a}{b}$ = ...

ⓐ 2 ⓑ $\frac{1}{2}$ ⓒ $\frac{1}{3}$ ⓓ $\frac{1}{4}$

(55) If $4x^2 = 9y^2$, then $\frac{x}{y}$ = ...

ⓐ $\frac{9}{4}$ ⓑ $\frac{3}{2}$ ⓒ $\pm\frac{2}{3}$ ⓓ $\pm\frac{3}{2}$

(56) If $\frac{a+2b}{a-b} = \frac{2}{3}$, then $\frac{b}{a}$ = ...

ⓐ $\frac{1}{8}$ ⓑ 8 ⓒ $\frac{-1}{8}$ ⓓ -8

(57) If $5a - 4b = 0$, then $\frac{a}{b}$ = ...

ⓐ $\frac{4}{5}$ ⓑ $\frac{5}{4}$ ⓒ $\frac{-4}{5}$ ⓓ $\frac{-5}{4}$

(58) If $\frac{5a - 7b}{8a + 11} = 0$, then $\frac{b}{a}$ = ………

ⓐ $\frac{5}{7}$ ⓑ $\frac{7}{5}$ ⓒ $\frac{-8}{7}$ ⓓ 0

(59) If $\frac{4}{x} = \frac{7}{y} = \frac{b}{y - x}$, then b = ………

ⓐ 3 ⓑ -3 ⓒ 11 ⓓ -11

(60) If $\frac{a}{3} = \frac{b}{8} = \frac{a + \frac{1}{2}b}{x}$, then x = …………

ⓐ 7 ⓑ 11 ⓒ 9 ⓓ 5

(61) If $\frac{a}{b} = \frac{c}{d} = m$ where $m \neq 0$, then $\frac{a \times c}{b \times d}$ = ………

ⓐ $2m^2$ ⓑ m^2 ⓒ m ⓓ 2m

(62) If $\frac{a}{5} = \frac{b}{7}$, then 7a – 5b + 3 = …………

ⓐ 3 ⓑ 7 ⓒ 5 ⓓ 2

(63) If $\frac{x}{5} = \frac{y}{4} = \frac{x + 2y}{k}$, then k = …………

ⓐ 9 ⓑ 14 ⓒ 13 ⓓ 8

(64) If $\frac{a}{4} = \frac{b}{5}$ and 2a + 3b = 46, then a = …………

ⓐ 2 ⓑ 4 ⓒ 5 ⓓ 8

(65) If $\frac{a}{b} = \frac{2}{3}$ and $\frac{a}{c} = \frac{4}{5}$, then b : c = ……………

ⓐ 3 : 4 ⓑ 5 : 6 ⓒ 6 : 5 ⓓ 4 : 3

(66) The positive middle proportional between a and b is ………

ⓐ $\sqrt{ab}$ ⓑ $-\sqrt{ab}$ ⓒ $\pm\sqrt{ab}$ ⓓ ab

(67) The third proportional of 9 and -12 is ....

a) -16 b) 8 c) 16 d) 108

(68) If 6 is the middle proportional between m and 2, then m = ...

a) 8 b) 12 c) 18 d) 36

(69) If $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then a = ...

a) 5×2^2 b) 40 c) 10 d) 2×5^3

(70) If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$, then $\frac{a}{d}$ = ...

a) 2 b) 4 c) 8 d) 16

(71) If a,2,4,b are in a continued proportional, then a + b = ...

a) 8 b) 1 c) 9 d) 7

(72) The middle proportional between (x-2) and (x+2) is ....

a) $\sqrt{x+2}$ b) $\sqrt{x^2-4}$ c) x^2-4 d) $\pm\sqrt{x^2-4}$

(73) The number that must be added to the numbers 1,3,6 to be in a continued proportional is ....

a) 1 b) 2 c) 3 d) 4

(74) If 7,x,$\frac{1}{y}$ are in a continued proportional, then x^2y = ...

a) 7 b) 14 c) 49 d) 1

(75) If y is the middle proportional between x and z, then $\frac{x}{z}$ = ...

a) $\frac{x^2}{y^2}$ b) $\frac{y^2}{z^2}$ c) $\frac{z^2}{y^2}$ d) $\frac{y^2}{x^2}$

(76) If $y = \frac{m}{x^2}$ where m is a constant $\neq 0$, then y α ....

a) x^2 b) x c) $\frac{1}{x}$ d) $\frac{1}{x^2}$

(77) If $x - 2y = 0$, then $x \propto$ ....

a) y b) y^2 c) $\frac{1}{y}$ d) $\frac{1}{y^2}$

(78) The relation that represents a direct variation between x and y is ....

a) $x y = 5$ b) $y = x + 2$ c) $\frac{x}{3} = \frac{4}{y}$ d) $\frac{x}{5} = \frac{y}{2}$

(79) If y varies inversely as x and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the proportion constant = ...

a) $\frac{3}{2}$ b) $\frac{2}{3}$ c) 2 d) 6

(80) If $x y^5$ = constant, then x varies inversely as ....

a) $\frac{1}{5}$ b) y^5 c) y d) y^2

(81) If $y \propto \frac{1}{\sqrt{x}}$, then x varies ....

a) directly as y^2

b) inversely as y^2

c) inversely as $\sqrt{y}$

d) inversely as y

(82) If $y = 3x - 6$, then $y \propto$ ....

a) x b) $\frac{1}{x}$ c) $x-2$ d) $3x-6$

(83) If $\frac{y+3}{y} = \frac{x+2}{x}$, $x \neq 0$, $y \neq 0$, then $y \propto$ ....

a) x b) $\frac{1}{x}$ c) $x+2$ d) $x+5$

(84) If $y - x = \frac{2}{y} - \frac{2}{x}$, $x \neq y$, then ....

a) $y \propto x + 1$ b) $y \propto x$ c) $y \propto \frac{1}{x}$ d) $y \propto \frac{1}{x^2}$

(85) If $9, 2x, \frac{1}{y^2}$ are proportional, then x y = ………

ⓐ $\frac{3}{2}$ ⓑ $\frac{-3}{2}$ ⓒ $\pm\frac{3}{2}$ ⓓ $\pm\frac{2}{3}$

(86) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$, then $\frac{ace}{bdf}$ = ………

ⓐ 3m ⓑ m^2 ⓒ m^3 ⓓ m

(87) If y α x and y = 2 as x = 4, then y = …… x

ⓐ 4 ⓑ 3 ⓒ 2 ⓓ $\frac{1}{2}$

(88) The mean of the values 7,3,6,9,5 is ………

ⓐ 3 ⓑ 6 ⓒ 4 ⓓ 12

(89) The range of the values 23,22,15,18,17 is ………

ⓐ 8 ⓑ 18 ⓒ 19 ⓓ 23

(90) If 67 is the greatest value and the range is 27, then the smallest value is ………

ⓐ 67 ⓑ 40 ⓒ 27 ⓓ 94

(91) The most common value of set of individuals is called ………

ⓐ median ⓑ range ⓒ mode ⓓ mean

(92) If the mean of the values 3k-3 , 3k-1 , 2k+1 , 2k+3 , 2k+5 is 13, then k = ………

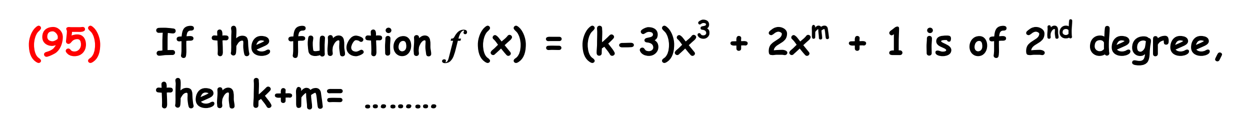
ⓐ -5 ⓑ 10 ⓒ 5 ⓓ $\frac{1}{5}$

(93) If the range of values 2,7,a,6 is 8 where a > 0, then a = ……

ⓐ 4 ⓑ 9 ⓒ -1 ⓓ 10

(94) If $(x - \overline{x})^2 = 28$ for the set 7 values, then σ = ……………

ⓐ 28 ⓑ 7 ⓒ 4 ⓓ 2



(95) If the function f (x) = $(k-3)x^3 + 2x^m + 1$ is of 2nd degree, then k+m= ....

ⓐ 5 ⓑ 3 ⓒ 2 ⓓ -5

(96) The difference between the greatest value and the smallest value is called ....

ⓐ median ⓑ mean ⓒ range ⓓ mode

(97) If the standard deviation for the values 5 , x+2 and 2y+1 is zero, then x + y = ...

ⓐ 10 ⓑ 5 ⓒ 15 ⓓ 0

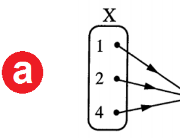
(98) The standard deviation for the values 7, 7, 7 is ....

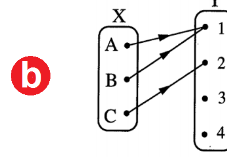
ⓐ 49 ⓑ 7 ⓒ 3 ⓓ 0

(99) If all individuals are equal, then ....

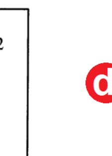
ⓐ X=0 ⓑ $\overline{X} = 0$ ⓒ σ=0 ⓓ mode=0

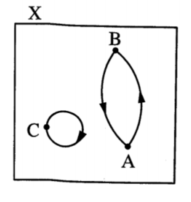
(100) Which of the following arrow diagrams does not represent a function

ⓐ 

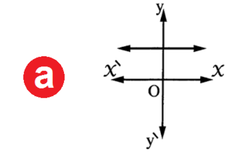
ⓑ 

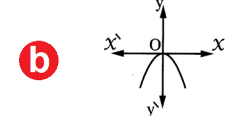
ⓒ 

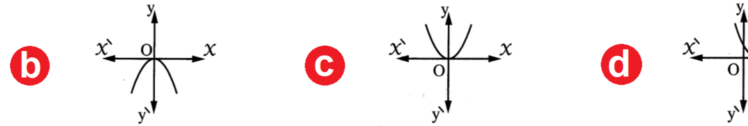


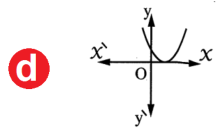
ⓓ 

(101) The graph of the function f where f (x) =x^2-2x+1 is the graph number ....

ⓐ 

ⓑ 

ⓒ 

ⓓ 

(102) If the curve of the function f where f (x) =x^2-a passes through the point (1,0), then a = ...

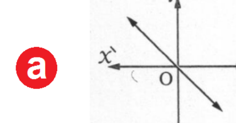
ⓐ ± 1 ⓑ -1 ⓒ 1 ⓓ zero

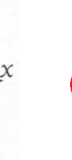


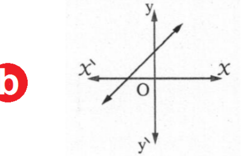
(103) If $f(x) = x^{k+3} + 2k$ is a quadratic function, then $f(2)$ = ...

ⓐ 1 ⓑ -1 ⓒ 2 ⓓ -2

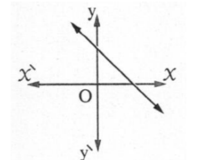
(104) The graph which represents the direct variation is number ......



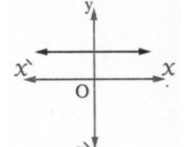












Essay problems:

(1) If X = {1,5,6} and Y = {5} and Z = {2,3}, Find:
(a) n(X × Z).
(b) (Y ∩ X) × (X - Y).

(2) If X × Y = {(2,3) , (2,6) , (2,7)}, Find:
(a) X and Y.
(b) Y^2.
(c) $n(X^2)$.

(3) If X = {2,3}, Y = {3,4} and Z = {4,5}, Find:
(a) Z × (X ∩ Y)
(b) (Z – Y) × X

(4) If (x+3 , 8) = (5 , 2^y), then find the value of x and y.

(5) If (x-2 , 9) = (5 , x+y), find the value of $\sqrt{3x + 2y}$.

(6) If (x^2, |y|) = (4,3) and (x,y) located in the 3^{rd} quadrant, then find x+y.

(7) If X={1,3,5} and Y={1,2,4,5,6} and R is a relation from X to Y where aRb means a+b=7 for a∈X and b∈Y. Write R, represent it by the arrow diagram, show that R is a function and write its range.

(8) If X={1,3,5} and R is a function on X where R={(a,3), (b,1), (1,5)}. Find the value of a+b.

(9) If $f(x)=2x^2-5x+2$, prove that $f(2)=f(\frac{1}{2})$

(10) If f is a function on X where X={3,4,5,6} and $f(3)=3$, $f(4)=5$, $f(5)=5$, $f(6)=5$. represent f by an arrow diagram, write f and find its range.

(11) If the straight line which represents the function $f(x)=ax+b$ intersects X-axis at (3,0) and Y-axis at (0,-3), find the value of a and b.

(12) If $(2a,5a) \in f$ where $f(x)=2x+5$, find the value of a and identify the intersection points of the straight line with the coordinates axes.

(13) If $f(x)=(3-a)x^2+(b+5)x+4$ is a constant function. Find the value of a+b.

(14) If the vertex of the curve of the function $f(x)=x^2-ax+3$ is (2,k). Find the value of a and k.

(15) Represent graphically the function $f(x)=4-x^2$, where $x\in[-3,3]$ and from the graph identify:
(a) The vertex.
(b) The equation of the axis of symmetry.
(c) The maximum or minimum value.

(16) Represent graphically the function $f(x)=x^2+2x+1$, where $x\in[-4,2]$ and from the graph identify:
(a) The vertex.
(b) The equation of the axis of symmetry.
(c) The maximum or minimum value.

(17) If $\frac{x-2y}{x+3y}=\frac{1}{3}$, find the value $\frac{y}{x}$.

(18) If $\frac{x}{y} = \frac{2}{3}$, find the value of $\frac{3x+2y}{6y-x}$.

(19) Find the number that if added to the two terms of the ratio 7:11 it becomes 2:3

(20) Find the number must be added to each of the numbers 3,5,8 and 12 to be proportional.

(21) Find the number if subtract its triple from the two terms of the ratio 49:69 it becomes 2:3.

(22) Find the number if we added its square to the two terms of the ratio 7:11 it becomes 4:5

(23) If $\frac{a+b}{b} = \frac{c+d}{d}$, prove that a, b, c and d are proportional.

(24) If $\frac{a}{b-a} = \frac{c}{d-c}$, prove that a, b, c and d are proportional.

(25) If a, b, c and d are proportional, prove that:

(a) $\frac{3a+c}{5a-2c} = \frac{3b+d}{5b-2d}$

(b) $\frac{a^2+b^2}{ab+cd} = \frac{a}{b}$

(c) $\frac{ac}{bd} = \left(\frac{a-c}{b-d}\right)^2$.

(26) If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, prove that $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$.

(27) If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$, find the value of x.

(28) If $\frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$, prove that $\frac{x+y}{a} = \frac{y+z}{b}$.

(29) If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, prove that

$$\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}.$$

(30) If $\frac{a+b}{4} = \frac{b+c}{5} = \frac{c+a}{7}$, prove that $\frac{a+b+c}{8} = \frac{a}{3}$.

(31) If a, 3, 9, b are in a continued proportion, find the value of a and b.

(32) If $\frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2}$, prove that b is a middle proportion between a and c where ac is a positive quantity.

(33) If b is a middle proportion between a and c, prove that:

(a) $\frac{a}{c} = \frac{b^2}{c^2}$.

(b) $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$.

(34) If Y varies directly as X and Y=20 as X=7, Find the relation between X and Y, then find the value of X as Y=4.

(35) If Y α X and Y=14 as X=42, Find:
(a) The relation between Y and X.
(b) The value of Y as X=60.

(36) If $y \,\alpha\, \frac{1}{x}$ and Y=3 as X=2, Find:
(a) The relation between Y and X.
(b) The value of Y as X=1.5

(37) If $\frac{a+2b}{6} = \frac{b+3c}{3}$, prove that a α c.

(38) If $x^2y^2 - 6xy + 9 = 0$, prove that $y \,\alpha\, \frac{1}{x}$.

(39) If $4x^2 + 9y^2 = 12xy$, prove that $y \propto x$.

(40) From the opposite table:

X	2	4	6
Y	6	3	2

(a) Determine the type of variation.
(b) Find the constant of variation.
(c) Find the value of y as x=3

(41) If y=z+5, $z \propto \frac{1}{x}$ and y=6 as x=2. Find the relation between x and y, then find the value of y as x=1

(42) Calculate the mean and the standard deviation of the following values:

(a) 15, 6, 8, 12, 4.

(b) 5, 6, 7, 8, 9.

(43) Calculate the standard deviation of the following frequency distributions:

(a)

Values	0	1	2	3	4	5
Frequency	9	15	17	25	20	14

(b)

Sets	0-	2-	4-	6-	8-
Frequency	1	5	9	3	2

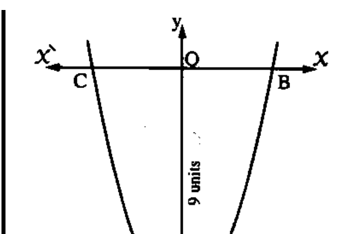
Drawn problems:

(1) The opposite figure represents the curve of the function f where $f(x) = x^2+k$. Find:

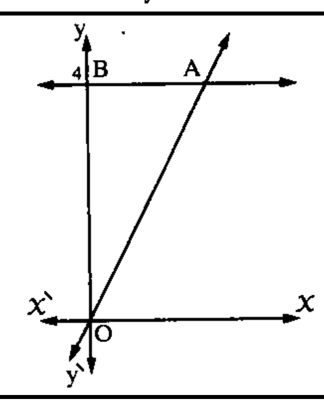
(a) The value of k.

(b) The coordinates of B and C.

(c) the area of triangle with vertices A,B,C

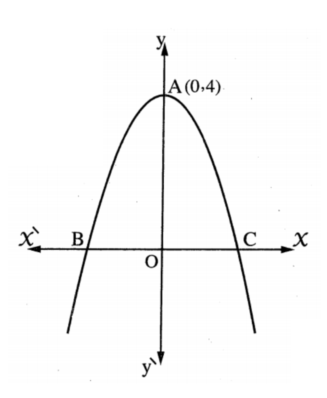


(2) The $\overleftrightarrow{AO}$ represents a linear function f where $f(x) = nx+k$ and the area of the Δ ABO is 4 square units. Find the value of n and k.



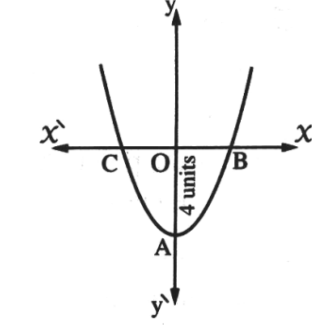
(3) The opposite figure represents the curve of the quadratic function f where $f(x) = 4-kx^2$, if the area of Δ ABC is 8 square units, Find:

(a) The value of k.
(b) The coordinates of B.
(c) The maximum or minimum value.
(d) The equation of the axis of symmetry.



(4) The opposite figure represents the curve of the function f where $f(x) = x^2-m$, Find:

(a) The value of m.
(b) The area of Δ ABC.



FOURTH: ACCUMULATIVE SKILLS ALGEBRA

(1)  {3} ⊂ ....

ⓐ (3,7) ⓑ]3,7] ⓒ]3,7[ⓓ {3,7}

(2) [2,7] – {2,7} = ...

ⓐ [1,6] ⓑ ∅ ⓒ]2,7[ⓓ {0}

(3) 2 567 approximated to the nearest five is ....

ⓐ 2 560 ⓑ 2 565 ⓒ 2 570 ⓓ 2 575

(4) $2^{2017} = 2^{2016} +$ ....

ⓐ 1 ⓑ 2 ⓒ 2016 ⓓ 2^{2016}

(5) If [-1,x] ∩ [y,5] = [2,3], then x^y = ...

ⓐ 8 ⓑ $\frac{1}{5}$ ⓒ 9 ⓓ -1

(6) When the side length of a square increases by the ratio 10%, then its area increases by the ratio .... %

ⓐ 10 ⓑ 15 ⓒ 20 ⓓ 21

(7) The ratio between the area of a square shaped region of side length x cm to the area of another square shaped region of side length 2x cm is ....

ⓐ 1:2 ⓑ X:4 ⓒ 1:4 ⓓ 4:1

(8) If x is an odd number, then the next odd number is ....

ⓐ x^2 ⓑ x^2+x ⓒ x+1 ⓓ x+2

(9) If M represents a negative number, which of the following represents a positive number?

ⓐ M^3 ⓑ M^2 ⓒ $2M$ ⓓ $M \div 2$

(10) Half of the number 2^{20} is ....

ⓐ 2^{10} ⓑ 1^{20} ⓒ 2^{19} ⓓ 1^{10}

(11) $3^{25} + 3^{25} + 3^{25} =$ ....

ⓐ 3^{75} ⓑ 3^{50} ⓒ 3^{26} ⓓ 3^{25}

(12) $3^x + 3^x + 3^x =$ ....

ⓐ 3^x ⓑ 3^{3x} ⓒ 3^{x+1} ⓓ 3^{x+3}

(13) $2^5 + 2^5 + 2^5 + 2^5 =$ ....

ⓐ 2^7 ⓑ 2^6 ⓒ 2^4 ⓓ 2^{20}

(14) If $x + y = 5$ and $x - y = \frac{1}{5}$, then $x^2 - y^2 =$ ....

ⓐ 125 ⓑ 1 ⓒ 25 ⓓ 5

(15) If $x + y = x\,y = 5$, then $x^2 y + y^2 x =$ ....

ⓐ 10 ⓑ 15 ⓒ 20 ⓓ 25

(16) If $(x - y)^2 = 20$ and $x^2 + y^2 = 10$, then $x\,y =$ ....

ⓐ 10 ⓑ 5 ⓒ -5 ⓓ 20

(17) If $1 < x < 3$, then $(3x-1) \in$ ....

ⓐ $[2,8[$ ⓑ $[2,8]$ ⓒ $]2,8[$ ⓓ $\{2,8\}$

(18) The S.S. of the inequality $5-3x > 11$ in R is ....

ⓐ $]-\infty,-2[$ ⓑ $]-2, \infty[$ ⓒ $]-\infty,-2]$ ⓓ $[-2,2]$

(19) The sum of the two square roots of the number $2\frac{1}{4}$ is ……

a) zero b) $\frac{3}{2}$ c) 3 d) $\frac{9}{4}$

(20) Four times the number 2^8 is ………

a) 2^{32} b) 8^8 c) 2^{10} d) 4^8

(21) If $x = \sqrt{3} + \sqrt{2}$ and $y = \frac{1}{\sqrt{3} + \sqrt{2}}$, then $(x + y)^2$ = ………

a) 8 b) 0 c) 9 d) 12

(22) If $2^x = \frac{1}{8}$, then x = …………

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) 3 d) -3

(23) If 100 grams of food contains 300 calories, then how many calories are there in 30 grams of the same food?

a) 90 b) 100 c) 1 000 d) 9 000

(24) A book contains 56 pages. How many times the number 5 appears in the pages serial of this book?

a) 6 b) 7 c) 12 d) 13

(25) If we put on one side of a road of length 12 km some light poles from the beginning to the end of the road, where the distance between each two consecutive poles is $\frac{1}{2}$ km, then the number of poles is ……………

a) 12 b) 24 c) 25 d) 23

(26) The decimal that lies between 0.07 and 0.08 is …………

a) 0.00075 b) 0.0075 c) 0.075 d) -0.75

(27) The square of double the number $\frac{1}{2}$ is ………

ⓐ $\frac{1}{4}$ ⓑ $\frac{1}{8}$ ⓒ 1 ⓓ 2

(28) $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = \frac{\cdots\cdot}{xy}$

ⓐ 2 ⓑ 3 ⓒ x+y+1 ⓓ x+y

(29) [1,6] – [1,6[= …………

ⓐ {1} ⓑ {9} ⓒ {6} ⓓ]1,6[

(30) $Z^- \cup N$ = ………

ⓐ ∅ ⓑ N ⓒ Z ⓓ R

FOURTH: ACCUMULATIVE SKILLS ALGEBRA

1.	D	2.	C	3.	B	4.	D
5.	C	6.	D	7.	C	8.	D
9.	B	10.	C	11.	C	12.	C
13.	A	14.	B	15.	D	16.	C
17.	C	18.	A	19.	A	20.	C
21.	D	22.	D	23.	A	24.	D
25.	C	26.	C	27.	C	28.	C
29.	C	30.	C				



FIRST: ALGEBRA

Choose the correct answer:

1.	D	2.	A	3.	A	4.	B
5.	C	6.	D	7.	A	8.	D
9.	A	10.	B	11.	A	12.	A
13.	C	14.	A	15.	C	16.	D
17.	C	18.	C	19.	C	20.	C
21.	B	22.	D	23.	A	24.	A
25.	C	26.	C	27.	C	28.	C
29.	A	30.	A	31.	A	32.	D
33.	D	34.	D	35.	B	36.	D
37.	B	38.	A	39.	A	40.	A
41.	C	42.	C	43.	C	44.	C
45.	A	46.	A	47.	B	48.	D
49.	A	50.	B	51.	D	52.	A
53.	A	54.	B	55.	D	56.	C
57.	A	58.	A	59.	A	60.	A
61.	B	62.	A	63.	C	64.	D
65.	C	66.	A	67.	C	68.	C
69.	B	70.	C	71.	C	72.	D
73.	C	74.	A	75.	B	76.	D
77.	A	78.	D	79.	C	80.	B
81.	B	82.	C	83.	A	84.	C
85.	C	86.	C	87.	D	88.	B
89.	A	90.	B	91.	C	92.	C
93.	D	94.	D	95.	A	96.	C
97.	B	98.	D	99.	C	100.	C
101.	D	102.	C	103.	C	104.	A